

Wage-Rise Contract and Mixed Duopoly with Price Competition

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Abstract

This paper investigates a mixed duopoly environment in which a private firm competes on price with a public firm. The following timing of actions is considered. In the first stage, each firm non-cooperatively decides whether to adopt a wage-rise contract as a strategic commitment device. If a firm adopts a wage-rise contract, then it chooses an output level and a wage premium rate, and agrees to pay each employee a wage premium uniformly if it actually produces more than the output level. This irreversible behaviour causes changes to the price-competing market environment of the second stage. The paper presents the equilibrium solution of the mixed duopoly model.

Keywords: Mixed market model, price competition, private firm, public firm, strategic commitment.

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Introduction

As is well known, mixed oligopolies can be found in common in developed and developing countries as well as in former communist countries. Private firms compete against public firms in many industries. Some of the typical examples are seen in banking, broadcasting, education, electricity, health care, home loans, life insurance, natural gas, rail, shipbuilding, telecommunications, and tobacco.

The first theoretical study of a public firm dates back to the 1960s (Merrill and Schneider, 1966). Since then, the analysis of mixed market models that incorporate public firms has received increasing attention and has been widely performed by many researchers (see, for instance, Delbono and Rossini, 1992; Delbono and Denicolò, 1993; Nett, 1994; Willner, 1994; Fjell and Pal, 1996; George and La Manna, 1996; White, 1996;

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Mujumdar and Pal, 1998; Pal, 1998; Pal and White, 1998; Poyago-Theotoky, 1998; Fjell and Heywood, 2002; Matsumura, 2003; Han and Ogawa, 2008; Ohnishi, 2008; Fernández-Ruiz, 2009; Lu and Poddar, 2009; Heywood and Ye, 2010; Zikos, 2010; Wang and Chen, 2011; Pal and Saha, 2014). These studies consider mixed oligopoly markets with quantity competition.

Only a few papers investigate mixed oligopoly markets with price competition as follows. Bös (1984) introduces public sector prices as government instruments into an integrated control-theory model of Mirrlees (1971), and demonstrates that separability does not usually exist between public pricing as an allocational instrument and income taxation as a distributional instrument. Cremer, Marchand and Thisse (1991) discuss a subgame perfect Nash equilibrium in a mixed oligopoly model where firms choose their location and price, and demonstrate that the case of mixed oligopoly with one public firm may be socially preferable to that of private oligopoly. Ogawa and Kato (2006) investigate simultaneous and sequential price setting in a homogenous product market with a private firm and a public firm, and demonstrate that the equilibrium price is higher when the private firm is the leader than when it is the follower. Bárcena-Ruiz (2007) examines a mixed duopoly in which firms choose whether to set prices sequentially or simultaneously and shows that they set prices simultaneously. Barcena-Ruiz and Garzón (2007) consider a two-stage mixed duopoly model in which each firm chooses its capacity and price, and show that if products are substitutes, then the public firm chooses over-capacity and the private firm chooses under-capacity. Ohnishi (2011) investigates a two-stage international mixed duopoly model where a state-owned public firm and a foreign private firm can provide lifetime employment as a strategic commitment, and demonstrates that the result of the analysis of international price-setting mixed duopoly competition with lifetime employment is beneficial only for the state-owned firm. Ohnishi (2012) focuses on the role that production subsidies play in a Bertrand mixed duopoly, and demonstrates that the results are the same as those of the existing Cournot mixed market literature. In addition, Ohnishi (2015) investigates a mixed duopoly model where a capitalist private firm and a state-owned public firm coexist, and demonstrates that introducing lifetime employment into the model of price-setting mixed duopoly may be beneficial for the state-owned public firm.

We examine the behaviour of a state-owned welfare-maximizing public firm and a profit-maximizing private firm in a price-setting mixed model. Each firm is allowed to adopt the wage-rise-contract policy (WRCP) (For details see Ohnishi, 2003, 2007). We consider a two-stage game with the following timing. In stage one, each firm simultaneously and non-cooperatively decides whether or not to adopt WRCP. If a firm adopts WRCP, then it chooses an output level and a wage premium rate, and agrees to pay each employee a wage premium uniformly if it actually produces more than the output level. In stage two, each firm simultaneously and non-cooperatively chooses its price. We present the equilibrium of the price-setting mixed duopoly model and as a result find that introducing WRCP into the analysis of the price-setting mixed duopoly model is beneficial only for the private firm.

The Model

We consider a mixed market environment composed of one state-owned public firm (firm 0) and one private firm (firm 1). The basic structure of the model is adopted from Bárcena-Ruiz (2007). Throughout this paper, subscripts 0 and 1 represent firm 0 and firm 1, respectively. In addition, when i and j are employed in an expression, they should be understood to represent 0 and 1 with $i \neq j$. We do not consider the possibility of entry or exit. The duopolists produce imperfectly substitutable goods. All the consumers are of the same type, and the representative consumer maximizes the following utility function:

$$U(q_0, q_1) - p_0q_0 - p_1q_1 \quad (1)$$

where q_i represents the quantity of good i and p_i is the price of good i . It is assumed that $U(q_0, q_1)$ is quadratic, strictly concave and symmetric with respect to q_0 and q_1 :

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{q_0^2 + 2bq_0q_1 + q_1^2}{2} \quad (2)$$

where $a \in (0, \infty)$ is constant and $b \in (0, 1)$ is a measure of the degree of substitutability between goods. The demand function is given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2} \quad (3)$$

Without loss of generality and for the sake of simplicity, it is assumed $b = 0.5$.

This market is modelled by means of the following two-stage game. In stage one, each firm simultaneously and non-cooperatively decides whether or not to adopt WRCP. If firm i adopts WRCP, then it chooses an output level $q_i^* \in [0, \infty)$ and a wage premium rate $w_i \in (0, \infty)$. In addition, firm i agrees to pay each employee a wage premium uniformly if it actually produces more than q_i^* . In stage two, each firm simultaneously and non-cooperatively chooses its price $p_i \in (0, \infty)$.

Therefore, firm i 's profit is given by

$$\pi_i = \begin{cases} (p_i - c)q_i & \text{if } q_i \leq q_i^* \\ (p_i - c)q_i - (q_i - q_i^*)w_i & \text{if } q_i \geq q_i^* \end{cases} \quad (4)$$

where $c \in (0, a)$ represents the total cost for each unit of output. We assume $c < a$ in order to assure that the production levels of firms are positive.

Moreover, social welfare (W) is the sum of consumer surplus (CS) and profits, and is given by

$$W = CS + \pi_0 + \pi_1 \quad (5)$$



Throughout this paper, we adopt subgame perfection as the equilibrium concept.

Results

In this section, we present the equilibrium of the model formulated in the previous section. Firstly, consider the case in which neither firm offers WRCP. We can derive the equilibrium values of prices and quantities from (4) and (5) as follows:

$$p_0^N = \frac{a+6c}{7}, \quad p_1^N = \frac{2a+5c}{7}$$

$$q_0^N = \frac{2(a-c)}{3}, \quad q_1^N = \frac{8(a-c)}{21}$$

Moreover, the profits, consumer surplus and social welfare can be obtained as follows:

$$\pi_0^N = \frac{2(a-c)^2}{21}, \quad \pi_1^N = \frac{16(a-c)^2}{147}$$

$$CS^N = \frac{62(a-c)^2}{147}, \quad W^N = \frac{92(a-c)^2}{147}$$

Secondly, consider the case in which only firm 0 offers WRCP. The equilibrium can be obtained as follows:

$$p_0^0 = \frac{a+6c+8w_0}{7}, \quad p_1^0 = \frac{2a+5c+2w_0}{7}$$

$$q_0^0 = \frac{2(a-c-2w_0)}{3}, \quad q_1^0 = \frac{8(a-c+w_0)}{21}$$

Moreover, the profits, consumer surplus and social welfare can be expressed as follows:

$$\pi_0^0 = \frac{2(a-c+8w_0)(a-c-2w_0)}{21}$$

$$\pi_1^0 = \frac{16(a-c+w_0)^2}{147}$$

$$CS^0 = \frac{2[31(a-c)^2 - 4w_0(16a-16c-13w_0)]}{147}$$

$$W^0 = \frac{4[23(a-c)^2 - w_0(3a-3c+26w_0)]}{147}$$

In this section, we present the equilibrium outcome of the mixed market model where both firm 0 and firm 1 are allowed to offer WRCP. However, the equilibrium outcomes in cases where only one firm can offer WRCP are one of our interests. Therefore, before presenting the equilibrium outcome in the bilateral case, we discuss the equilibrium outcomes in the unilateral cases.

Proposition 1: Suppose that only firm 0 can offer WRCP. Then there exists a unique equilibrium in which firm 0 does not offer WRCP.

Proof: We compare W^0 with W^N :

$$W^0 - W^N = \frac{-4w_0(3a-3c+26w_0)}{147} < 0$$

Thus Proposition 1 is true. *QED*

Firm 0 seeks to maximize social welfare. Therefore, firm 0 will adopt WRCP if social welfare increases by doing so, while firm 0 will not adopt WRCP if social welfare decreases by doing so. We explain the intuition behind Proposition 1. When firm 0 offers WRCP, it chooses q_0^* and w_0 , and agrees to pay each employee a wage premium uniformly if it actually produces more than q_0^* . Therefore, the adoption of WRCP by firm 0 raises its marginal cost of production, and hence increases p_0 . Increasing p_0 increases p_1 because of strategic complements. Increasing prices decreases the total market output and social welfare because of the downward sloping demand, and thus firm 0 has no incentive to offer WRCP.

Thirdly, consider the case in which only firm 1 offers WRCP. The equilibrium values of prices and quantities can be obtained as follows:

$$p_0^1 = \frac{a+6c+2w_1}{7}, \quad p_1^1 = \frac{2a+5c+4w_1}{7}$$

$$q_0^1 = \frac{2(a-c)}{3}, \quad q_1^1 = \frac{4(2a-2c-3w_1)}{21}$$

Moreover, we have the following levels of the profits, consumer surplus and social welfare:

$$\pi_0^1 = \frac{2(a-c)(a-c+2w_1)}{21}$$

$$\pi_1^1 = \frac{4(2a-2c-3w_1)(2a-2c+4w_1)}{147}$$

$$CS^1 = \frac{2[31(a-c)^2 - 6w_1(5a-5c-2w_1)]}{147}$$

$$W^1 = \frac{4[23(a-c)^2 - 6w_1(a-c+w_1)]}{147}$$

We can now state the following proposition.

Proposition 2: Suppose that only firm 1 can offer WRCP. Then there exists a unique equilibrium in which firm 1 offers WRCP. At equilibrium, firm 1 earns a higher profit than in the Bertrand game without WRCP, whereas social welfare is lower than in the Bertrand game without WRCP.

Proof: First we compare π_1^1 with π_1^N :

$$\pi_1^1 - \pi_1^N = \frac{8w_1(a-c-6w_1)}{147}$$

In the first stage, firm 1 can choose $w_1 < (a-c)/6$ and hence $\pi_1^N < \pi_1^1$.

Next, we compare W^1 with W^N :

$$W^1 - W^N = \frac{-24w_1(a-c+w_1)}{147} < 0$$

Thus Proposition 2 is proved. *QED*

Firm 1 seeks to maximize its own profit. Therefore, firm 1 will adopt WRCP if its profit increases by doing so, while firm 1 will not adopt WRCP if its profit decreases by doing so. The intuition behind Proposition 2 can be explained as follows. When firm 1 offers WRCP, it chooses q_1^* and w_1 , and agrees to pay each employee a wage premium uniformly if it actually produces more than q_1^* . Therefore, the adoption of WRCP by firm 1 raises its marginal cost of production, and hence increases p_1 . Increasing p_1 increases p_0 because of strategic complements. Increasing p_0 increases π_1 because of substitute goods, and thus firm 1 offers WRCP.

Fourthly, consider the case in which both firms offer WRCP. The equilibrium can be derived as follows:

$$p_0^B = \frac{a+6c+8w_0+2w_1}{7}, \quad p_1^B = \frac{2a+5c+2w_0+4w_1}{7}$$

$$q_0^B = \frac{2(a-c-2w_0)}{3}, \quad q_1^B = \frac{4(2a-2c+2w_0-3w_1)}{21}$$

Moreover, the profits, consumer surplus and social welfare can be expressed as follows:

$$\pi_0^B = \frac{2(a-c-2w_0)(a-c+8w_0+2w_1)}{21}$$

$$\pi_1^B = \frac{8(a-c+w_0+2w_1)(2a-2c+2w_0-3w_1)}{147}$$

$$CS^B = \frac{2[31(a-c)^2 - 2(32aw_0 + 15aw_1 - 32cw_0 - 15cw_1 - 26w_0^2 - 6w_0w_1 - 6w_1^2)]}{147}$$

$$W^B = \frac{4[23(a-c)^2 - 3aw_0 - 6aw_1 + 3cw_0 + 6cw_1 - 26w_0^2 - 6w_0w_1 - 6w_1^2]}{147}$$

The main result of this study is described by the following proposition.

Proposition 3: In the price-setting mixed duopoly model, there exists a unique equilibrium in which only firm 1 offers WRCP.

Proof: In stage one, each firm simultaneously and non-cooperatively decides whether or not to adopt WRCP. In stage two, each firm simultaneously and non-cooperatively chooses its price. Our equilibrium concept is subgame perfection and all information in the model is common knowledge.

We now compare W^B with W^1 :

$$W^B - W^1 = \frac{-4w_0(3a-3c+26w_0+6w_1)}{147} < 0$$

From the preceding results, $W^0 < W^N$ and $W^B < W^1$, so that firm 0 never offers WRCP. On the other hand, since $\pi_1^N < \pi_1^1$, firm 1 offers WRCP. Proposition 3 is proved. *QED*

Propositions 1 and 3 indicate that the best firm 0 can do is not to offer WRCP whether or not firm 1 does so. On the other hand, Propositions 2 and 3 means that the best firm 1 can do is to offer WRCP whether or not firm 0 does so. The intuition behind Proposition 3 can be as follows. The adoption of WRCP by firm i increases p_i . Moreover, increasing p_i increases p_j because of strategic complements. Increasing p_j increases π_i because of substitute goods. Therefore, firm 1 has an incentive to offer WRCP. In addition, increasing prices decreases the total market output and social welfare because of the downward sloping demand. Thus, firm 0 has no incentive to offer WRCP.

Concluding Remarks

We have studied the behaviour of a state-owned welfare-maximizing public firm and a profit-maximizing private firm in a price-setting mixed model. First, we have considered the case in which only the public firm is allowed to offer WRCP. In this case,

we have demonstrated that there is an equilibrium where the public firm does not offer WRCP. Second, we have considered the case in which only the private firm is allowed to offer WRCP, and have demonstrated that there is an equilibrium where the private firm offers WRCP. Third, we have shown that if both firms are allowed to offer WRCP, there is a unique equilibrium in which the private firm offers WRCP while the public firm does not. As a result, we see that introducing WRCP into the analysis of the price-setting mixed duopoly model is beneficial only for the private firm.

Finally, we would like to describe the policy implications of this study. The adoption of WRCP by a firm increases its marginal cost of production, and hence decreases its optimal output. Moreover, the total market output decreases. Hence consumer surplus and social welfare are lower than in the Bertrand game without WRCP. Therefore, if private and public firms compete on price with each other, then governments that wish to increase social welfare should not adopt a policy that increases firms' marginal costs. As a result, we find that such governments should adopt an industrial policy that decreases firms' marginal costs and promotes competition among firms.

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