

Quantity Precommitment and Cournot and Bertrand Models with Complementary Goods

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Abstract

This paper investigates Cournot and Bertrand duopoly models with complementary goods, where firms can enter into lifetime employment contracts with their respective workers as a strategic device. The paper treats the following four cases: ‘Cournot competition with strategic complements’, ‘Cournot competition with strategic substitutes’, ‘Bertrand competition with strategic substitutes’ and ‘Bertrand competition with strategic complements’. The paper presents the equilibrium outcomes of the four cases. In addition, it is shown that lifetime employment is beneficial for firms in the cases with strategic complements.

Keywords: Cournot model, Bertrand model, complementary goods, lifetime employment

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Introduction

This paper considers a market situation in which firms are allowed to use lifetime employment as a strategic commitment device (see Ohnishi, 2001, 2002). If a firm legally enters into a lifetime employment contract with its workers, then its wage cost sinks and its marginal cost decreases.

Kreps and Scheinkman (1983) use capacity investment, which functions just as well as lifetime employment used in this paper. They examine the subgame perfect equilibrium of price-setting duopoly competition with homogeneous goods, and show that the unique equilibrium coincides with the Cournot solution. Yin and Ng (1997) show that two-stage price-setting duopoly competition with substitute goods, following a simultaneous

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endogenous choice of capacity, yields the Cournot solution. Ohnishi (2006) investigates the respective equilibrium outcomes of a price-setting lifetime-employment-contract game with substitute goods and a price-setting lifetime-employment-contract game with complementary goods, and demonstrates that in each game, the equilibrium coincides with the Bertrand outcome with no lifetime employment. In addition, Ohnishi (2012) examines the respective equilibrium outcomes of the two games of a quantity-setting duopoly game with substitute goods and a quantity-setting duopoly game with complementary goods, and finds that the introduction of lifetime employment into the analysis of quantity-setting game with complementary goods increases economic welfare.

We study quantity-setting and price-setting models with complementary goods, where duopolists can enter into lifetime employment contracts with their respective workers as a strategic commitment. We treat the following four cases: ‘Cournot competition with strategic complements’, ‘Cournot competition with strategic substitutes’, ‘Bertrand competition with strategic substitutes’, and ‘Bertrand competition with strategic complements’. We present the subgame perfect equilibria when duopolists are allowed to adopt lifetime employment by using quantity-setting and price-setting models. As a result of this analysis, it is shown that lifetime employment is beneficial for duopolists in the cases with strategic complements.

The Quantity-Setting Model and Its Equilibrium Outcomes

In this section, we formulate the quantity-setting model with complementary goods and discuss its equilibrium outcomes. There are two firms, designated firm 1 and firm 2. In the balance of this paper, when i and j are used in an expression, they represent 1 and 2 with $i \neq j$. Firm i 's profit is

$$\pi_i(q_i, q_j) = p_i(q_i, q_j)q_i - v_i q_i \quad (1)$$

where p_i represents firm i 's inverse demand function, q_i is firm i 's output, and v_i is firm i 's constant marginal cost.

The quantity-setting game is played in the following sequence. In stage 1, each firm i noncooperatively decides whether to use lifetime employment as a strategic commitment device. If firm i uses lifetime employment, then it chooses its quantity q_i^* and enters into a lifetime employment contract with the workers necessary to achieve q_i^* . In stage 2, each firm i noncooperatively chooses its actual quantity q_i . At the end of stage 2, the market opens and firm i sells its actual output q_i . Therefore, firm i 's profit is shown as follows:

$$\hat{\pi}_i(q_i^*, q_i, q_j) = \begin{cases} \pi_i(q_i, q_j) & \text{if } q_i \geq q_i^* \\ \pi_i(q_i, q_j) - (q_i^* - q_i)r_i & \text{if } q_i \leq q_i^* \end{cases} \quad (2)$$

where $r_i \in (0, v_i]$ represents firm i 's wage cost per unit of output. If firm i chooses q_i^* and enters into a lifetime employment contract with all of the workers necessary to achieve q_i^* , then its marginal cost is affected by the lifetime employment contract. Hence, firm i 's

marginal cost has a discontinuity at $q_i = q_i^*$. Given q_j , firm i solves its profit maximization problem with respect to q_i .

If firm i 's marginal cost of output equals v_i , then a usual way to define its quantity reaction function is

$$R_i^v(q_j) = \arg \max_{\{q_i \geq 0\}} \pi_i(q_i, q_j) \quad (3)$$

and if firm i 's marginal cost of output equals $v_i - r_i$, then its quantity reaction function is illustrated by

$$R_i^{v-r}(q_j) = \arg \max_{\{q_i \geq 0\}} [\pi_i(q_i, q_j) + r_i q_i] \quad (4)$$

Therefore, if firm i chooses q_i^* and provides lifetime employment, then we can define its quantity best response as follows:

$$R_i(q_j) = \begin{cases} R_i^{v-r}(q_j) & \text{if } q_i < q_i^* \\ q_i^* & \text{if } q_i = q_i^* \\ R_i^v(q_j) & \text{if } q_i > q_i^* \end{cases} \quad (5)$$

The Cournot-Nash equilibrium can be defined as quantity levels (q_1^C, q_2^C) where $q_i^C \in R_i(q_j^C)$.

It is assumed that there exists a unique Cournot-Nash equilibrium in $0 < q_i < \infty$. Moreover, the following assumptions are added.

Assumption 1 (differentiability): $p_i(q_i, q_j)$ is twice continuously differentiable with $\partial p_i / \partial q_i < 0$ (downward-sloping demand) and $\partial p_i / \partial q_j > 0$ (complementary goods).

Assumption 2 (concavity of profit function): $\partial^2 \pi_i / \partial q_i^2 < 0$.

Assumption 3 (stability): If $(R_i(q_j), q_j) \in \mathbb{R}_{++}^2$, then $0 < |R_i'(q_j)| < 1$.

These are standard assumptions in Cournot duopoly games except complementary goods. Throughout this paper, our solution concept is subgame perfection. As usual, we apply backward induction.

We present here the following lemmas.

Lemma 1: If firm i chooses q_i^* and uses lifetime employment as a strategic commitment device, then at equilibrium $q_i = q_i^*$.

Proof: See Ohnishi (2015, Lemma 1).

Lemma 2: The provision of lifetime employment by firm i increases its profit-maximizing quantity.

Proof: See Ohnishi (2015, Lemma 2).

We now explore the following two cases.

Case 1: $\partial R_i / \partial q_j > 0$

Case 2: $\partial R_i / \partial q_j < 0$

We treat these cases in turn.

Case 1

This is the case of strategic complements in which goods are complements. Figure 1 depicts both firms' reaction curves.

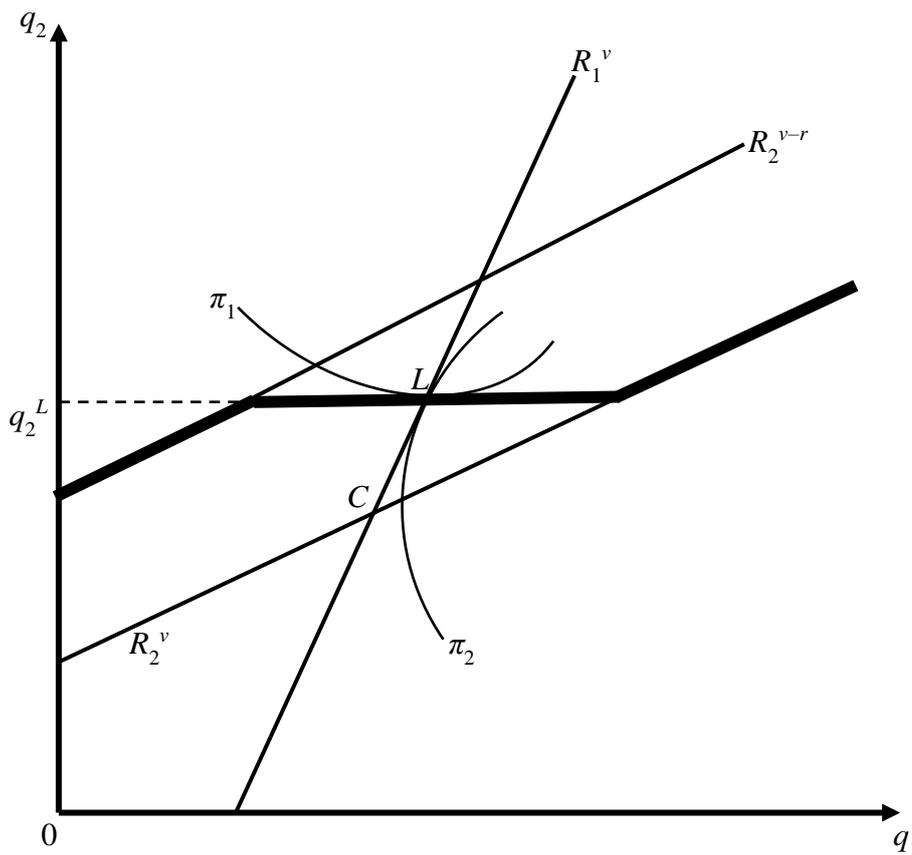


Figure 1 The Cournot game with strategic complements

In this figure, R_i^v represents firm i 's reaction curve when its marginal cost of output equals v_i , R_i^{v-r} is firm i 's reaction curve when its marginal cost of output equals $v_i - r_i$, and π_i is firm i 's isoprofit curve extending through point L .

Both R_i^v and R_i^{v-r} are upward sloping. If neither firm uses lifetime employment as a strategic commitment device, then the unique solution is at C .

Assume that only firm 2 provides lifetime employment to its workers. From Lemma 2, we can understand that the provision of lifetime employment by firm i increases its optimal quantity and shifts its quantity reaction curve to the right. Therefore, if firm 2 chooses q_2^L and provides lifetime employment to its workers, then its marginal cost of output has a discontinuity at $q_2 = q_2^L$ and its quantity reaction curve becomes the thick kinked line as drawn in this figure. The solution is determined in a Cournot-Nash fashion. The quantity reaction curves cross at L . From this figure, we can understand that firm 2's profit is higher at point L than point C . The solution of this case is characterized in the following proposition.

Proposition 1: In the quantity-setting game with $\partial p_i / \partial q_j > 0$ and $\partial R_i / \partial q_j > 0$, there exists a unique subgame perfect Nash equilibrium in which at least one firm uses lifetime employment as a strategic commitment device. At the equilibrium, both firms obtain higher profits than in the Cournot game with no lifetime employment.

Proof: See Ohnishi (2012, Propositions 3 and 4).

Proposition 1 implies that each duopolist can improve its profit by using lifetime employment as a strategic device.

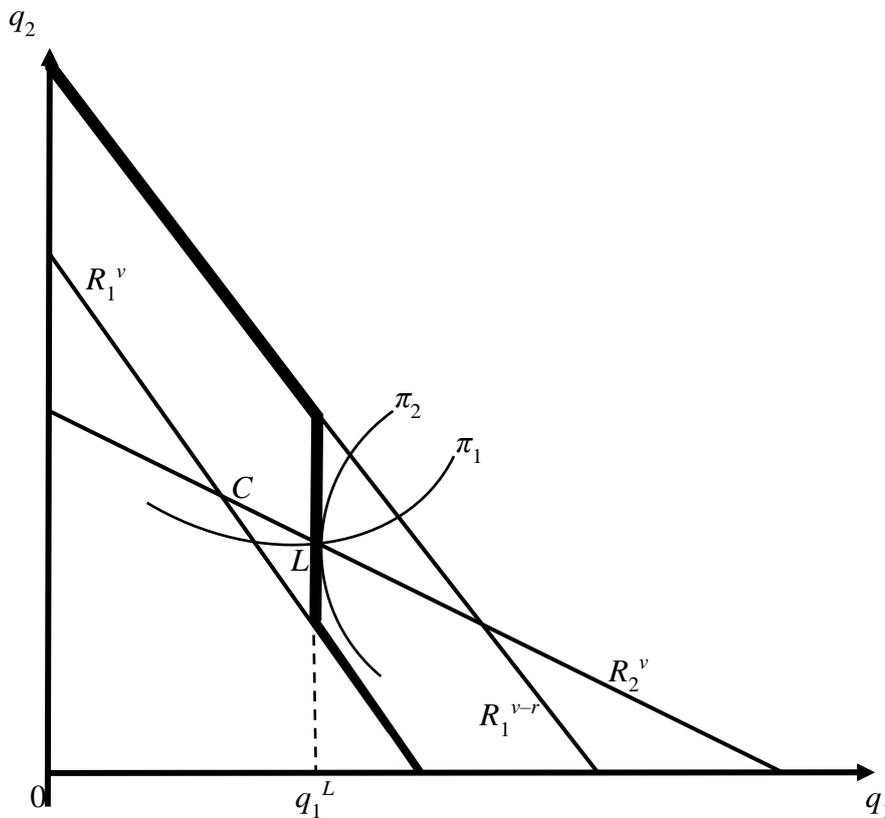


Figure 2 The Cournot game with strategic substitutes

Case 2

We next analyse the case of strategic substitutes in which goods are complements. Figure 2 depicts both firms' reaction curves.

Both R_i^v and R_i^{v-r} slope downwards. If neither firm provides lifetime employment to its workers, then the unique Cournot equilibrium occurs at point C . Lemma 2 states that the provision of lifetime employment by firm i increases its optimal quantity and shifts its quantity reaction curve to the right. Therefore, if firm 1 chooses q_1^L and provides lifetime employment to its workers, then its marginal cost of output has a discontinuity at $q_1 = q_1^L$ and its quantity reaction curve becomes the thick kinked line as drawn in this figure. The firms choose quantities in a Cournot fashion. The quantity reaction curves cross at point L . From this figure, we can understand that firm 1's profit is lower at point L than point C .

We now characterize the equilibrium of Case 2 in the following proposition.

Proposition 2: In the quantity-setting game with $\partial p_i / \partial q_j > 0$ and $\partial R_i / \partial q_j < 0$, there exists a unique subgame perfect Nash equilibrium that coincides with the Cournot output with no lifetime employment.

Proof: First of all, we prove that if firm i provides lifetime employment to its workers, then firm i 's profit is lower than its Cournot profit with no lifetime employment. Lemma 2 states that the provision of lifetime employment by firm i increases its profit-maximizing output. From Assumption 1, if we differentiate π_j with respect to q_i , then it give us $\partial\pi_j/\partial q_i = (\partial p_j/\partial q_i) q_j > 0$. Hence, firm j 's profit exceeds its Cournot profit with no lifetime employment.

Increasing firm i 's output increases firm j 's amount demanded because of complementary goods. Firm j 's profit increases even at its Cournot output. In firm j 's optimal strategy, its output decreases because of strategic substitutes.

Decreasing firm j 's output decreases firm i 's amount demanded because of complementary goods. From Assumption 1, if we differentiate π_i with respect to q_j , then it give us $\partial\pi_i/\partial q_j = (\partial p_i/\partial q_j) q_i > 0$. Decreasing firm j 's output decreases firm i 's profit. Therefore, neither firm has an incentive to unilaterally deviate from the equilibrium with no lifetime employment. Thus the proposition is valid. *QED*

Proposition 2 indicates that neither duopolist can improve its profit by providing lifetime employment. Thus, the unique outcome coincides with the Cournot solution with no lifetime employment.

The Price-Setting Model and Its Equilibrium Outcomes

In this section, we formulate the price-setting model with complementary goods and discuss its equilibrium outcomes. Firm i 's profit is

$$\pi_i(p_i, p_j) = p_i q_i(p_i, p_j) - v_i q_i(p_i, p_j) \quad (6)$$

where p_i represents firm i 's price, and q_i is firm i 's demand function.

The price-setting game runs as follows. In stage 1, each firm i noncooperatively decides whether to provide lifetime employment to its workers. If firm i uses lifetime employment as a strategic device, then it chooses q_i^* and enters into a lifetime employment contract with the workers necessary to achieve q_i^* . In stage 2, each firm i noncooperatively chooses its price p_i . At the end of stage 2, the market opens and firm i sells its actual quantity $q_i(p_i, p_j)$. Therefore, firm i 's profit changes as follows:

$$\hat{\pi}_i(q_i^*, p_i, p_j) = \begin{cases} \pi_i(p_i, p_j) & \text{if } q_i(p_i, p_j) \geq q_i^* \\ \pi_i(p_i, p_j) - (q_i^* - q_i(p_i, p_j))r_i & \text{if } q_i(p_i, p_j) \leq q_i^* \end{cases} \quad (7)$$

Given p_j , firm i must solve its profit maximization problem with respect to p_i . If firm i 's marginal cost of output equals v_i , then a usual way to define its price reaction function is

$$R_i^v(p_j) = \arg \max_{\{p_i \geq 0\}} \pi_i(p_i, p_j) \quad (8)$$

and if firm i 's marginal cost of output equals $v_i - r_i$, then its price reaction function is

$$R_i^{v-r}(p_j) = \arg \max_{\{p_i \geq 0\}} [\pi_i(p_i, p_j) + r_i q_i(p_i, p_j)] \quad (9)$$

Therefore, if firm i chooses q_i^* and enters into a lifetime employment contract with its workers, then we can define its price best response changes as follows:

$$R_i(p_j) = \begin{cases} R_i^{v-r}(p_j) & \text{if } q_i(p_i, p_j) < q_i^* \\ q_i^* & \text{if } q_i(p_i, p_j) = q_i^* \\ R_i^v(p_j) & \text{if } q_i(p_i, p_j) > q_i^* \end{cases} \quad (10)$$

The Bertrand-Nash equilibrium can be defined as price levels (p_1^B, p_2^B) where $p_i^B \in R_i(q_j^B)$.

It is assumed that there is a unique Bertrand-Nash equilibrium in $0 < p_i < \infty$. Moreover, the following assumptions are introduced.

Assumption 4 (differentiability): $q_i(p_i, p_j)$ is twice continuously differentiable with $\partial q_i / \partial p_i < 0$ (downward-sloping demand) and $\partial q_i / \partial p_j < 0$ (complementary goods).

Assumption 5 (concavity of profit function): $\partial^2 \pi_i / \partial p_i^2 < 0$.

Assumption 6 (stability): If $(R_i(p_j), p_j) \in \mathbb{R}_{++}^2$, then $0 < |R_i'(p_j)| < 1$.

These are standard assumptions in Bertrand games except complementary goods. We prove here the following lemma.

Lemma 3: The provision of lifetime employment by firm i lowers its profit-maximizing price.

Proof: From (7), we understand that the provision of lifetime employment by firm i never increases its marginal cost of output. If firm i 's marginal cost of output equals v_i , then the first-order condition for profit maximization is

$$q_i + p_i \frac{\partial q_i}{\partial p_i} - v_i \frac{\partial q_i}{\partial p_i} = 0 \quad (11)$$

However, if firm i 's marginal cost of output equals $v_i - r_i$, the first-order condition for profit maximization is

$$q_i + p_i \frac{\partial q_i}{\partial p_i} - v_i \frac{\partial q_i}{\partial p_i} + r_i \frac{\partial q_i}{\partial p_i} = 0 \quad (12)$$

where $r_i > 0$ and $\partial q_j / \partial p_i < 0$ (from Assumption 1). Hence, the sign of $q_i + p_i (\partial q_j / \partial p_i) - v_i (\partial q_j / \partial p_i)$ needs to be plus in order to satisfy (12). This completes the proof. *QED*

We handle the following two cases.

Case 3: $\partial R_i / \partial p_j < 0$

Case 4: $\partial R_i / \partial p_j > 0$

We now examine these two cases in turn.

Case 3

We explore the case of strategic substitutes in which goods are complements. Figure 3 depicts both firms' reaction curves. Both R_i^v and R_i^{v-r} are downward sloping. Assume that firm 2 unilaterally uses lifetime employment as a strategic commitment device. From Lemma 4, we can understand that the provision of lifetime employment by firm i lowers its optimal price and shifts its price reaction curve to the left.

Therefore, if firm 2 chooses q_2^L and provides lifetime employment to its workers, then its marginal cost of output has a discontinuity at $q_2 = q_2^L$ and its price reaction curve becomes the thick kinked line as drawn in this figure. The price reaction curves cross at point L . From this figure, we see that firm 2's profit is lower at point L than point B .

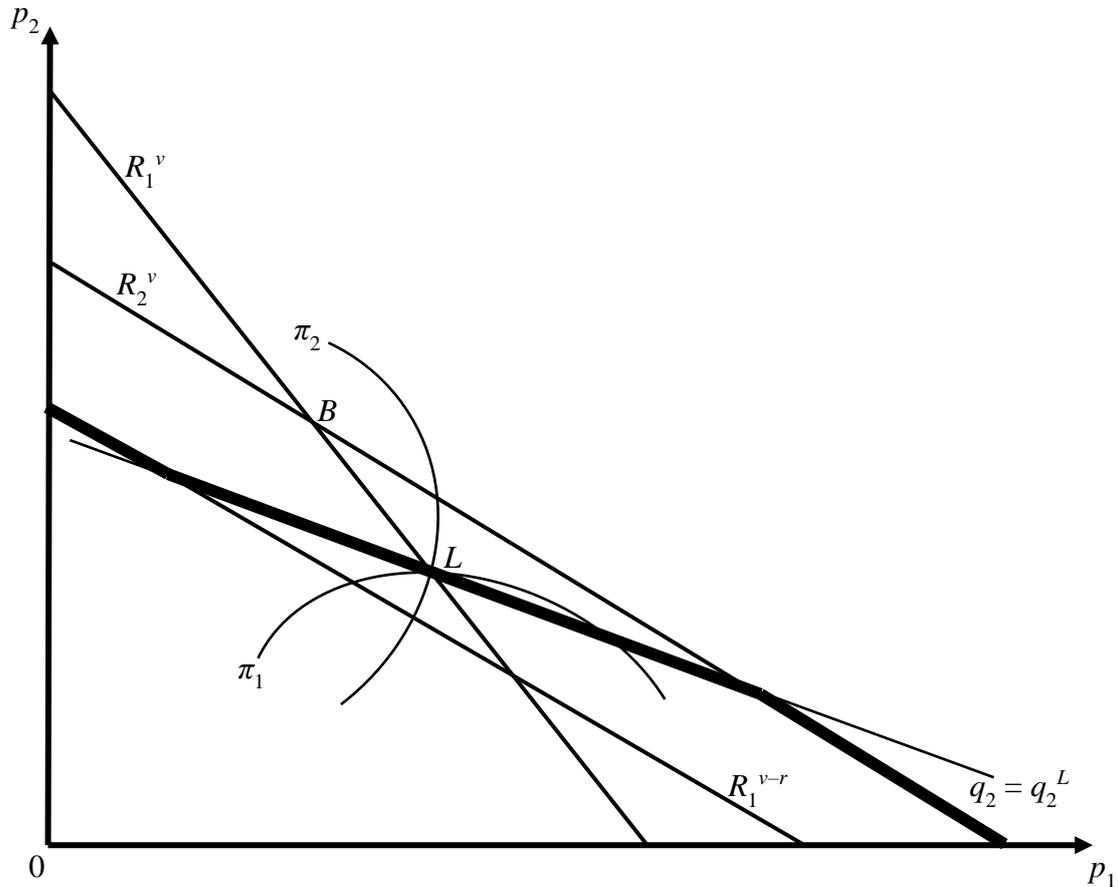


Figure 3 The Bertrand game with strategic substitutes

The solution of Case 3 is characterized in the following proposition.

Proposition 3: In the price-setting game with $\partial q_i / \partial p_j < 0$ and $\partial R_i / \partial p_j < 0$, there exists a unique subgame perfect Nash equilibrium that coincides with the Bertrand solution with no lifetime employment.

Proof: See Ohnishi (2006, Lemma 3 and Proposition 2).

Proposition 3 implies that there are no firms that uses lifetime employment as a strategic device. That is, neither duopolist is able to increase its profit by providing lifetime employment. Therefore, the unique equilibrium coincides with the Bertrand solution with no lifetime employment.

Case 4

Case 4 is the case of strategic complements in which goods are complements. Figure 4 depicts both firms' reaction curves. Both R_i^v and R_i^{v-r} slope upwards. If nether firm provides lifetime employment, then the unique solution remains at B .

Assume that only firm 1 provides lifetime employment to its workers. From Lemma 3, we see that the provision of lifetime employment by firm i lowers its optimal price and shifts its price reaction curve to the left. Therefore, if firm 1 chooses q_1^L and provides lifetime employment, then its marginal cost of output has a discontinuity at $q_1 = q_1^L$ and its price reaction curve becomes the thick kinked line as drawn in this figure. The price reaction curves cross at L . The firms choose prices in a Bertrand-Nash fashion. From this figure, we can understand that firm 1's profit is higher at point L than point B .

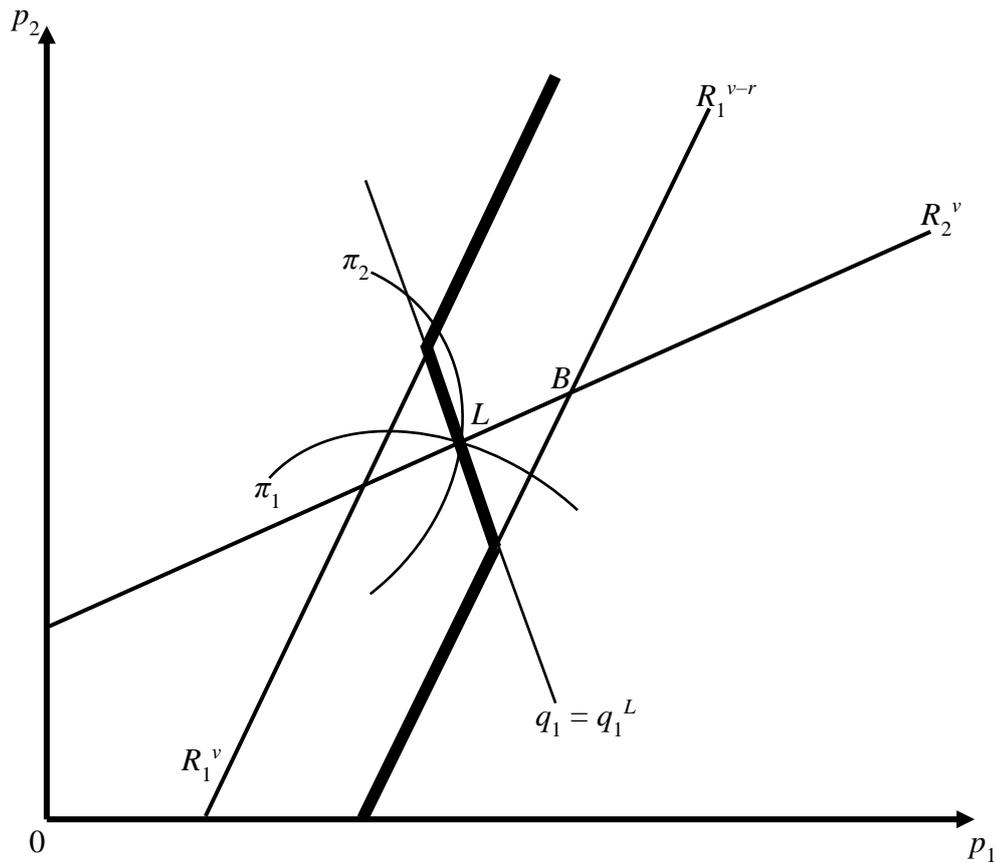


Figure 4 The Bertrand game with strategic complements

We present here the following lemma.

Lemma 4: In the price-setting game with $\partial q_i / \partial p_j < 0$ and $\partial R_i / \partial p_j > 0$, if one firm unilaterally uses lifetime employment as a strategic device, then each firm's profit exceeds its Bertrand profit with no lifetime employment.

Proof: Assume that firm i unilaterally uses lifetime employment as a strategic device. Lemma 3 states that the provision of lifetime employment by firm i lowers its profit-maximizing price. Decreasing firm i 's price increases firm j 's amount demanded because of complementary goods. Firm j 's profit increases even at its Bertrand price. Firm j 's optimal strategy must yield at least this profit. Hence, firm j 's profit exceeds its Bertrand profit with no lifetime employment. In firm j 's optimal strategy, its price drops because

of strategic complements. Decreasing firm j 's price increases firm i 's amount demanded because of complementary goods. If $q_i = q_i^*$, then decreasing firm j 's price increases firm i 's profit. Lemma 1 states that at equilibrium $q_i = q_i^*$. Thus, firm i 's profit also exceeds its Bertrand profit with no lifetime employment. *QED*

We now present the equilibrium of Case 4 in the following proposition.

Proposition 4: In the price-setting game with $\partial q_i / \partial p_j < 0$ and $\partial R_i / \partial p_j > 0$, there exists a unique subgame perfect Nash equilibrium in which at least one firm uses lifetime employment as a strategic commitment device. At the equilibrium, both firms enjoy higher profits than in the Bertrand game with no lifetime employment.

Proof: Lemma 4 states that the unilateral lifetime employment solution generates a higher profit for each firm than at the Bertrand solution with no lifetime employment. Hence, there exists no equilibrium in which neither firm provides lifetime employment because cycling of choices is impossible.

From Lemma 4, the rest of the results hold if only one firm uses lifetime employment as a strategic commitment device. There is a bilateral lifetime employment equilibrium if the bilateral lifetime employment profits exceed the unilateral lifetime employment profits. Thus, Proposition 4 follows. *QED*

Proposition 4 implies that lifetime employment is beneficial for both duopolists.

Conclusion

We have presented the equilibrium outcomes of quantity-setting and price-setting models with complementary goods. In addition, it has been shown that lifetime employment can be beneficial for firms in the games with strategic complements. If a game is played in strategic complements, then at least one firm provides lifetime employment in a non-cooperative solution. Lifetime employment enables both firms to get more in a non-cooperative game. Therefore, we can say that it facilitates tacit collusion. As a result of this analysis, we find that tacit collusion is facilitated in the games with strategic complements.

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