Endogenous Timing in a Price-Setting Mixed Duopoly with a Foreign Competitor

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Abstract
This paper considers mixed duopoly games where a state-owned public firm and a foreign private firm compete in price. The public firm aims to maximize the un-weighted sum of consumer surplus and its own profit. The paper examines a desirable role (either leader or follower) of the public firm, an effect of eliminating the foreign firm and an endogenous role in price-setting mixed duopoly by adopting the observable delay game. Consequently, the paper shows that the unique equilibrium of price-setting international mixed competition is quite different from that of quantity-setting international mixed competition.

Keywords: Price competition, endogenous timing, mixed market, foreign private firm.

Introduction
As is well known, international mixed oligopolies are found in many developed, developing and former communist countries. Public firms compete against foreign private firms in many industries, such as banking, life insurance, steel, shipbuilding, and tobacco. In the tobacco industries of France, Italy, Russia, Spain, Austria, Turkey, India, China, Japan etc. we can find real-world examples in which public firms compete or competed against foreign private firms such as Philip Morris and R. J. Reynolds.

Matsumura (2003) considers a quantity-setting mixed duopoly where a state-owned public firm competes with a foreign private firm. He first investigates a desirable role (either leader or follower) of the public firm and shows that it should be the leader.

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Second, he investigates an effect of eliminating the foreign firm and shows that eliminating the foreign firm never improves domestic social surplus in quantity-setting mixed duopoly. Third, he examines an endogenous role in quantity-setting mixed duopoly by adopting the observable delay game of Hamilton and Slutsky (1990) and demonstrates that the public firm becomes the leader in the unique equilibrium. In addition, Bárcena-Ruiz and Sedano (2011) investigate the endogenous order of moves in a mixed duopoly with price competition, assuming that the objective function of the public firm can give a different weight to consumer surplus than to producer surplus and that firms produce heterogeneous goods.

We consider a mixed duopoly where a state-owned public firm and a foreign private firm compete in price. Ohnishi (2010) examines both domestic and international mixed markets with price setting and compares the two equilibrium outcomes domestic and international mixed markets with price competition. Bárcena-Ruiz and Sedano (2011) examine endogenous timing in a mixed duopoly where the private firm is foreign-owned and the public firm aims to maximize the weighted sum of consumer surplus and its own profit. However, Bárcena-Ruiz and Sedano do not consider the case of un-weighted welfare where firms product substitute products.

We examine a setting where the public firm aims to maximize the un-weighted sum of consumer surplus and its own profit and goods are substitute goods. We investigate a desirable role of the public firm, an effect of eliminating the foreign firm and an endogenous role in price-setting mixed duopoly by adopting the observable delay game. The results of this study are compared with those of quantity-setting mixed duopoly games with a foreign competitor.

**The Model**

There is an industry composed of one foreign private firm (firm 1) and one domestic public firm (firm 2), producing imperfectly substitutable goods. In the balance of this paper, subscripts 1 and 2 denote firm 1 and firm 2, respectively. In addition, when i and j are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with i ≠ j. We do not consider the possibility of entry or exit. Following Barcena-Ruiz and Garzón (2007), we assume that all consumers are of the same type and the representative consumer maximizes the following utility function:

\[ U(q_1, q_2) = p_1q_1 - p_2q_2, \]  

(1)

where \( q_i \) denotes the amount of good \( i \) and \( p_i \) is its price. The function \( U(q_1, q_2) \) is quadratic, strictly concave and symmetric in \( q_1 \) and \( q_2 \):

\[ U(q_1, q_2) = a(q_1 + q_2) - \frac{q_1^2 + 2bq_1q_2 + q_2^2}{2}, \]  

(2)

where \( 0 < a \) and \( 0 < b < 1 \). The demand function is given by
where $b$ is a measure of the degree of substitutability between products. Without loss of generality and for the sake of simplicity, we assume $b = 0.5$.

Each firm’s profit is given by

$$\pi_i = (p_i - c)q_i,$$

where $c$ denotes the total cost for each unit of output. We assume $0 < c < a$ to assure that the production levels of firms are positive. Firm 1 aims to maximize its own profit.

Domestic social surplus, defined as the sum of domestic consumer surplus and firm 2’s profit, is given by

$$S = CS + \pi_2,$$

where $CS = 2[(p_1)^2 - p_1p_2 + (p_2)^2 + a(a - p_1 - p_2)]/3$. Firm 2 aims to maximize domestic social surplus.

Formally, the game is as follows. In the first stage, each firm simultaneously and independently chooses $e_i \in \{2, 3\}$, where $e_i$ indicates when to decide the non-negative price $p_i$. That is, $e_i = 2$ implies that firm $i$ decides in the second period, and $e_i = 3$ implies that it decides in the third stage. At the end of the first stage, each firm observes $e_1$ and $e_2$. In the second stage, firm $i$ choosing $e_i = 2$ selects its price $p_i$ in this stage. In the third stage, firm $i$ choosing $e_i = 3$ selects its price $p_i$ in this stage. At the end of the game, the market opens and each firm sells its output at $p_i$.

**Fixed Timing Games**

In this section, we examine two Stackelberg and one Bertrand duopoly games of fixed timing. First, we consider the Stackelberg game where firm 2 is the leader. Firm 2 chooses $p_2$ and firm 1 chooses $p_1$ after observing $p_2$. The superscript $L$ denotes the equilibrium outcome of the game where firm 2 is the leader. We present the public leader equilibrium values of prices, outputs, profits, domestic consumer surplus and domestic social surplus:

$$p_1^L = \frac{9a + 23c}{30}, \quad p_2^L = \frac{16c - a}{15}, \quad q_1^L = \frac{14(a - c)}{45}, \quad q_2^L = \frac{41(a - c)}{45},$$

$$\pi_1^L = \frac{98(a - c)^2}{1350}, \quad \pi_2^L = \frac{41(c - a)(a - c)}{675}, \quad CS^L = \frac{817(a - c)^2}{1350}, \quad \delta^L = \frac{49(a - c)^2}{90}.$$

Note that firm 2’s output is unambiguously larger than firm 1’s. Also note that firm 2’s profit is negative.
Second, we consider the Stackelberg game where firm 2 is the follower. Firm 1 chooses \( p_1 \) and firm 2 chooses \( p_2 \) after observing \( p_1 \). The superscript \( F \) denotes the equilibrium outcome of the game where firm 2 is the follower. We present the foreign private leader equilibrium values of prices, outputs, profits, domestic consumer surplus and domestic social surplus:

\[
\begin{align*}
    p_1^F &= \frac{a + 3c}{4}, &
    p_2^F &= c, &
    q_1^F &= \frac{a - c}{3}, &
    q_2^F &= \frac{5(a - c)}{6}, \\
    \pi_1^F &= \frac{(a - c)^2}{12}, &
    \pi_2^F &= 0, &
    CS^F &= \frac{13(a - c)^2}{24}, &
    S^F &= \frac{13(a - c)^2}{24}.
\end{align*}
\]

Note that firm 2’s price equals marginal cost and hence its profit is zero. Also note that firm 2’s output is two and a half as large as firm 1’s.

Third, we consider the Bertrand game where each firm simultaneously and independently chooses \( p_i \). It happens that prices, outputs, profits and surpluses in this game are identical with those in the Stackelberg game where firm 2 is the follower. In the remainder of this paper, the superscript \( B \) denotes the equilibrium outcome of the Bertrand game.

We now state the following proposition.

Proposition 1: (i) \( S^L > S^B \) and (ii) \( S^B = S^F \).

Proposition 1 means that firm 2 should be the leader. Proposition 1 (i) is the same as that of quantity-setting international mixed duopoly. On the other hand, Proposition 1 (ii) is different from that of quantity-setting international mixed duopoly.

We can also make a remark on the profit of firm 1.

Lemma 1: (i) \( \pi_1(p_1^F, p_2^F) = \pi_1(p_1^B, p_2^B) \), (ii) \( \pi_1(p_1^B, p_2^B) > \pi_1(p_1^L, p_2^L) \), and (iii) \( \pi_1(p_1^F, p_2^F) > \pi_1(p_1^L, p_2^L) \).

Lemma 1 indicates that firm 1 does not have a second mover advantage. This result is contrary to that of quantity-setting international mixed duopoly.

Before discussing the equilibrium outcome in the endogenous timing game, we consider whether eliminating a foreign competitor improves domestic social surplus.

Proposition 2: Let \( S^M \) denote domestic social surplus when firm 2 is the monopolist. Then \( S^F > S^M \).

We present the public monopoly equilibrium values of prices, outputs, profits and surpluses: \( p_2^M = c, q_2^M = a - c, \pi_2^M = 0, CS^M = (a - c)^2 / 2 \) and \( S^M = (a - c)^2 / 2 \). Thus, the assertion of Proposition 2 is clear. Since \( S^L > S^B = S^F \), Proposition 2 means that eliminating a foreign competitor never improves domestic social surplus. This result is the same as with that of quantity-setting international mixed duopoly.
Equilibrium in the Observable Delay Game

In this section, we now find the subgame perfect Nash equilibrium of the observable delay game formulated in Section 2. The main result of this study is described by the following proposition.

Proposition 3: In the unique equilibrium, \( e_1 = e_2 = 2 \).

Proof. First, we prove that in equilibrium \( e_1 = 2 \). Suppose that \( e_1 = 3 \), then the outcome becomes \((p_1, p_2) = (p_1^L, p_2^L)\). If \( e_2 = 2 \), then the outcome becomes \((p_1, p_2) = (p_1^B, p_2^B)\). Lemma 1 (ii) states that firm 1 must choose \( e_1 = 2 \). Suppose that \( e_2 = 3 \). If \( e_1 = 3 \), then the outcome becomes \((p_1, p_2) = (p_1^L, p_2^L)\). If \( e_1 = 2 \), then the outcome becomes \((p_1, p_2) = (p_1^F, p_2^F)\). Lemma 1 shows that \( \pi_1(p_1^F, p_2^F) = \pi_1(p_1^B, p_2^B) > \pi_1(p_1^L, p_2^L) \). Hence, firm 1 always chooses \( e_1 = 2 \) irrespective of the choice of firm 2.

Next, we prove that in equilibrium \( e_2 = 2 \). Suppose that \( e_1 = 3 \). If \( e_2 = 3 \), then the outcome becomes \((p_1, p_2) = (p_1^B, p_2^B)\). If \( e_2 = 2 \), then the outcome becomes \((p_1, p_2) = (p_1^L, p_2^L)\). Proposition 1 (i) states that firm 2 should choose \( e_2 = 2 \). Suppose that \( e_1 = 2 \). If \( e_2 = 3 \), then the outcome becomes \((p_1, p_2) = (p_1^F, p_2^F)\). If \( e_2 = 2 \), then the outcome becomes \((p_1, p_2) = (p_1^B, p_2^B)\). Proposition 1 shows that \( S^L > S^B = S^F \). Therefore, firm 2 always chooses \( e_2 = 2 \) irrespective of the choice of firm 1. Thus, the unilateral equilibrium becomes \( e_1 = e_2 = 2 \). Q.E.D.

Proposition 3 means that firm 2 cannot play the role of the Stackelberg leader. This result is quite different from that of Matsumura (2003).

Conclusion

We have considered a mixed duopoly where a state-owned public firm and a foreign private firm compete in price. We have first investigated whether the public firm should be the leader or follower and have shown that it should be the leader. Second, we have investigated an effect of eliminating the foreign firm and have shown that eliminating the foreign firm never improves domestic social surplus in price-setting international mixed duopoly. Third, we have examined an endogenous role in price-setting mixed duopoly by adopting the observable delay game and have demonstrated that the public firm cannot become the leader in the unique equilibrium. As a result, we have found that the unique equilibrium of price-setting international mixed competition is quite different from that of quantity-setting international mixed competition.

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References


