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Studying Substitution of Factors and Economies of Scale Using the Composite Cost Function in Fars Cement Factory

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Abstract

It is important to analyze the relationship between inputs and economies in the production due to high share of added value and great contribution of cement industry in national production and construction projects. In this paper, returns to scale, elasticities of substitution, function coefficients and economies of scale in Fars Cement Company were extracted and analyzed using the composite cost function and dual cost approach. The iterated nonlinear seemingly unrelated regression (NLSUR) method from 2002 to 2012 and quarterly data with Limdep8 Software were used in order to extract the cost function. Allen cross partial elasticity results for each pair of inputs showed that labor is a substitution for capital and other services. Capital input and other services are complementary. In addition, demand price elasticities indicate inelasticity of production factors against changes in price of those factors. According to research results, production of cement in Fars Cement Company resulted in ascending economies of scale and returns to scale.

Keywords: Elasticities of substitution, composite cost function, returns to scale, iterated nonlinear seemingly unrelated regressions.

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Introduction

Nowadays, it is essential to measure efficiency of various economic components in order to identify the factors affecting those economic factors, particularly in developing

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countries such as Iran considering faded economic boundaries and intensified competition in the global arena.

Industrial development is considered as one necessary condition for economic development. The experts specially paid attention to cement industry in the process of industrial development among other industries since it is fundraising, it creates occupational opportunities, it is consumes fuel and electricity and it creates an added value in the industrial sector. It is also considered as one of the most important institutions in the construction sector due to national demand in both terms of reconstruction and completion of infrastructure projects.

Given the above-mentioned material, understanding the economic efficiency of cement industry was always considered by policy makers and economic planners. This is not possible unless considering both cost structure and the technology to reduce costs and consequently increasing the profits. Finally, planning is not possible in this industry unless those matters in the above were observed.

The present study aimed to investigate and determine the indices of economies of scale, returns to scale and technological changes in Fars Cement Company. The composite cost function model is provided at first. Then, an overview of literature is provided. Then, the models and data analysis are discussed. In the end, summary and conclusion are presented.

Theoretical Principles

In this section, theoretical principles of the relationship between production and cost, various cost functions and the composite cost function are presented. Then, a model to estimate the firm's cost function based on composite cost function and the advantages compared to other types of cost functions are analyzed.

Composite cost function

Composite cost function (PB_c) was first introduced by Pulley and Braunstein (1992). The composite cost function is a combination of a quadratic structure for multiple outputs and a log-quadratic structure for input prices. This structure is desirable to investigate the properties of multi-product technologies. One advantage of this function compared to standard Translog function (normal) and generalized Translog function lies in the fact that composite cost function can be used for production output levels; i.e. zero output levels. This function is as follows.

$$LnC = Ln\left(\alpha_0 + \sum\alpha_i\, q_i + \frac{1}{2}\sum\sum\alpha_{ij}\, q_i\, q_j + \sum\sum\delta_{ik}\, q_i\, Lnr_k\right) + \beta_0 + \sum\sum u_{ik}\, q_i\, Lnr_k + \sum\beta_k\, Lnr_k + \frac{1}{2}\sum\sum\beta_{kl}\, Lnr_k Lnr_l + \varepsilon \quad (1)$$

where ε represents the error expression, C denotes total cost of production, i = 1 ... m, q_i denotes the outputs, r_k, and r_l represents the input prices.

Due to properties of a well-behaved cost function, following constraints were applied to equation (1) (Mutairi and Burney, 2002).

A- Hypothesis of homogeneity

$$\sum_{i=1}^{n} \alpha_i = 1 , \sum_{i=1}^{n} \beta_{ij} = \sum_{j=1}^{n} \beta_{ji} = \sum_{i=1}^{n} \delta_{iQ} = \sum_{i=1}^{n} \beta_{ti} = 0 \quad (2)$$

B- Hypothesis of symmetry

$$\beta_{ij} = \beta_{ji} \quad i \neq j \quad i,j = 1,2 \ldots n \quad (3)$$

The cost share equations were used in order to evaluate cost elasticities of factors. Those cost share equations were derived from the cost equation.

Then, symmetry and homogeneity hypotheses were applied to the desired function using Lemma Shepherd. Cost share function of each production factor was calculated using equation (4).

$$S_l = \frac{\partial \text{Lnc}(p,q)}{\partial \text{Ln} r_l} = \frac{\partial c}{\partial p_l} \frac{p_l}{c} = x_l \frac{p_l}{c} \quad (4)$$

On the other hand, one share equation should be eliminated, so that variance - covariance matrix determinant of confounding components would not become zero since total share costs is$(\sum_{i=1}^{3} S_i = 1)$. It is possible to eliminate each one of the equations in experimental work in order to make the best estimation. In this study, the share of capital was selected and eliminated from total cost. Hence, the latter condition as well as symmetry and homogeneity hypotheses were applied to the cost function and demand share equations. Then, cost estimation in this study is as follows.

$$Ln\left(\frac{C}{r_k}\right) = Ln\left(\alpha_0 + \alpha_q q + \frac{1}{2}\alpha_{qq} q^2 + \delta_{ql} q Ln\left(\frac{r_l}{r_k}\right) + \delta_{qs} q Ln\left(\frac{r_s}{r_k}\right) + +\varphi_t T + \varphi_{tt} T^2\right) + \beta_l Ln\left(\frac{r_l}{r_k}\right) + \beta_s Ln\left(\frac{r_s}{r_k}\right) + \beta_{ls} Ln\left(\frac{r_l}{r_k}\right) Ln\left(\frac{r_s}{r_k}\right) + \frac{1}{2}\beta_{ss} Ln\left(\frac{r_s}{r_k}\right)^2 + \frac{1}{2}\beta_{ll} Ln\left(\frac{r_l}{r_k}\right)^2 \quad (5)$$

In the above equation, C represents total cost of production, q denotes product of the firm or industry, r_l represents labor input prices, r_k denotes capital input prices, r_s represents other services input prices and T represents time trend variable.

$$S_l = (\delta_{ql} q)\left(\alpha_0 + \alpha_q q + \frac{1}{2}\alpha_{qq} q^2 + \delta_{ql} q Ln\left(\frac{r_l}{r_k}\right) + \delta_{qs} q Ln\left(\frac{r_s}{r_k}\right) + \varphi_t T + \varphi_{tt} T^2\right)^{-1} + \beta_l + \beta_{ll} Ln\left(\frac{r_l}{r_k}\right) + \beta_{ls} Ln\left(\frac{r_s}{r_k}\right) \quad (6)$$

In equation (6), S_l represents the share of labor input.

$$S_s = (\delta_{qs} q)\left(\alpha_0 + \alpha_q q + \frac{1}{2}\alpha_{qq} q^2 + \delta_{ql} q Ln\left(\frac{r_l}{r_k}\right) + \delta_{qs} q Ln\left(\frac{r_s}{r_k}\right) + \varphi_t T + \varphi_{tt} T^2\right)^{-1} + \beta_s + \beta_{ss} Ln\left(\frac{r_s}{r_k}\right) + \beta_{ls} Ln\left(\frac{r_l}{r_k}\right) \quad (7)$$

In equation (7), S_s represents the share of other services input.

The above system was estimated using iterated nonlinear seemingly unrelated regression (NLSUR) with Limdep8 Software. Then, the cost function indices such as

Allen-Uzawa Elasticities of Substitution, price elasticity, economies of scale, returns to scale and technological changes were estimated. As a result, production factors could be extracted.

It is suitable to use Allen elasticity of substitution (AES) in order to determine substitution capability of various factors of production.

Allen elasticity of substitution using Lemma Shepherd is represented in equations (8) and (9).

$$AES_{kl} = \frac{\beta_{kl}+S_k S_l}{S_k\, S_l} \qquad k \neq l \tag{8}$$

$$AES_{kk} = \frac{\beta_{kk}+S_k^2-S_k}{S_k^2} \qquad k = l \tag{9}$$

In the above equation, S_k and S_l respectively represent share of l and k factors while β_{kl} denotes cross-price inputs coefficient in the composite cost function.

Direct and cross elasticities of demand are presented in equations (10) and (11).

$$\eta_{kl} = \frac{\beta_{kl}+S_k S_l}{S_k} \qquad k \neq l \tag{10}$$

$$\eta_{kk} = \frac{\beta_{kk}+S_k^2-S_k}{S_k} \qquad k = l \tag{11}$$

In equations (10) and (11), η_{kl} represents cross-price elasticity between factors of production while η_{kk} denotes price elasticity, S_k and S_l respectively represent share of k and l factors and β_{kl} cross-price inputs coefficient in the cost function. Another method lies in calculating the cost elasticity with respect to product as equation (12).

$$ECS = \varepsilon_{cqi} = \frac{\partial Lnc}{\partial Lnq_i} \tag{12}$$

Larger (smaller) than one unit cost elasticity with respect to product indicates losses (benefits) to scale.

According to Baumol et al (1982), returns to scale index is equal to inverse of cost elasticity with respect to product.

$$RTS = \left[\frac{\partial LnTC}{\partial LnQ}\right]^{-1} = \frac{1}{\varepsilon_{cq}} \tag{13}$$

In the above equation, RTS denotes returns to scale.

In order to determine the impact of technical changes in cost function, time trend variable T was included in the model. Then, the growth rate was calculated according to the following equation.

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$$\varepsilon_{c_t} = (\varphi_t T + 2\varphi_{tt}\mathrm{T}\ )\left(\alpha_0 + \alpha_q q + \frac{1}{2}\alpha_{qq}q^2 + \delta_{ql}qLnr_l + \delta_{qs}qLnr_s + \delta_{qk}qLnr_k + \varphi_t T + \varphi_{tt}T^2\right)^{-1} \quad (14)$$

In equation (14), ε_{c_t} represents technical changes, T denotes time trend variable, q represents production, r_l, r_k and r_s denote prices of production factors.

If either $-\frac{\partial Ln(c)}{\partial(Trend)} > 0$ or $\varepsilon_{c_t} < 0$, there would be technological advances during the period studied.

Bloch and others (2001) analyzed cost structure of telephone services in Australia using a composite cost function and time series data from 1926 to 1991. According to the research results, Australian telephone services led to economies of scale; however, no economies of diversification were found in this sector.

Mutairi and Burney (2002) examined both factor substitution and economies of scale in Kuwait Oil Industry in their study. Composite time series data from 1976 to 1996 were used in order to estimate the function. The results indicated that applied production structure was not homothetic. It was found out that the former function was due to the effect of scale.

Piacenza and Vannoni (2004) evaluated the cost function in a sample of Italian Public Sector (a combination of gas, water and electricity) using a multi-product composite model from 1994 to 1996. Generalized Least Squares (GLS) method was used in order to estimate the parameters. The results showed that properties of multi-product composite cost function (PB_c) were more compatible with observed data compared to Standard Translog (ST), Generalized Translog (GT) and Separable Quadratic (SQ).

Bottasso et al (2010) examined the cost structure for a sample of England and Wales Water and Wastewater Industry in their study. For this purpose, composite cost function was used for a period from 1995 to 2005. The results showed that there were economies of scale while there were no economies of diversity.

Negahban (2002) conducted a study titled as the cost function of Iranian Telecommunication Corporation and calculation of productivity growth. He studied the cost structure of Iranian Telecommunication Corporation considering urban and suburban calls (wire telephone devices). The research results showed that capital and labor factors were inelastic to price changes while raw materials and energy were elastic in long term. On the other hand, labor, raw materials and energy were elastic in short-term.

Karimi (2009) examined the cost structure of Shiraz Water Supply and Wastewater Company in a study. For this purpose, cost function of the firm was estimated using Translog function with partial derivatives of Taylor second series as well as cost share equations with iterated seemingly unrelated regression method. Calculating Allen - Uzawa elasticity of substitution indicated that elasticities of substitution between each one of the production factors and mean capital had a positive sign, which suggested a substitution relationship between these factors.

The experimental results

Describing the data used

Required data was extracted using prime cost reports, financial statements, seasonal reports submitted to the department of accounting and finance and other units in Fars Cement Company. Total production cost in Fars Cement Company includes cost of salaries, costs of capital and cost of other services. Capital costs include cost of depreciation and maintenance of machinery and equipment. Labor costs include all payments to labor, whether a fixed salary, overtime payment, shift work payments, rewards, bonuses, service end benefits and other benefits paid to the employers in compliance with labor laws and social security organizations. The cost of other services include costs of transportation, rental of machinery, rental of location, petrol, diesel and oil consumed by transporting vehicles as well as other sectors in cement production.

All costs were converted to actual ones using the Producer Price Index published by Central Bank with respect to the constant price in 2007. Cement was the main product produced in Fars Cement Company during the period under study. All produced items were entered as tons in the cost function. Since the necessary data to calculate cost of capital was not found, the former was calculated using the index of user cost of capital based on the following equation (Ghorbani, 2012).

P_k = Long-term bank deposit interest rates + Depreciation Rate - Inflation Rate (15)

In the above equation, the inflation rate and the interest rate of long-term bank deposits were extracted from Central Bank Website. Furthermore, depreciation rate was obtained according financial statements of the Fars Cement Company.

The price of labor is the average of wages, salaries and benefits paid to each employer in each period. The adjusted wage and wage payments were divided to the number of employees in production sector to calculate the average wage according to equation (16).

$$\text{Pl} = \frac{labor\ wages\ (in\ Rial)}{\text{the number of employees}} \qquad (16)$$

Price of other services was calculated by dividing adjusted cost of other services to the volume of produced cement per ton.

Cost function estimation results

Total cost model and cost share equations were extracted using iterated nonlinear regression method with LIMDEP8 Software Package. Prior to model estimation, Augmented Dickey Fuller Test (ADF) was used to perform unit root test of variables.

According to the results, $Ln\left(\frac{C}{r_k}\right)$, $Ln\left(\frac{r_l}{r_k}\right)$ and $Ln\left(\frac{r_s}{r_k}\right)$ variables were at static level while q, S_t, S_k and S_S were at unstatic level. Unstatic variables became static with a difference.

Due to non-uniform static class variables, the remaining expressions of each one of the estimated equations were evaluated after estimating the model. The remaining expressions were at static level. Therefore, the model could be estimated without risk of spurious regression model.

Table 1 - Estimated cost function parameters

Parameter	Estimated values of the parameters	t-statistic	Standard error	Probability
α_0	0.4045	4.636	0.0872	0.000
α_q	1.4314	4.946	0.2894	0.000
α_{qq}	-1.1611	-3.085	0.3797	0.0022
δ_{ql}	-0.1015	-1.584	0.0640	0.1031
δ_{qs}	0.1573	6.616	0.0237	0.000
δ^*_{qk}	-0.0557	-	-	-
φ_t	-0.0112	-6.172	0.0018	0.000
φ_{tt}	0.3002	1.962	0.1530	0.0498
β_l	0.6118	11.998	0.0509	0.000
β_s	0.1024	3.794	0.0269	0.0001
β^*_k	0.2857	-	-	-
β_{ls}	-0.0835	-5.476	0.0152	0.000
β_{ll}	0.0917	3.709	0.2472	0.0002
β^*_{lk}	-0.0081	-	-	-
β_{ss}	0.2026	13.342	0.0151	0.000
β^*_{sk}	-0.1191	-	-	-
β^*_{kk}	0.1273	-	-	-
Number of observation = 44 Log likelihood = 65.9999 McElroy R-squared for the system = 0.975 Cost function R^2 = 0.6874 Other services R^2 = 0.4730 Labor share R^2 = 0.4113				

* Indirectly estimated parameters using homogeneity constraints

In addition, Lagrange multiplier statistic was used to check for contemporaneous correlation between the equations. Critical value was calculated as 10.33. This value was compared with the critical statistic at 95% level of confidence with two degrees of freedom (7.81). Since the calculated statistic value was larger than the critical value, null hypothesis (H0) was rejected. Then, there is a contemporaneous correlation between confounding components of equations. As a result, the model could be estimated using NLSUR method.

It should be noted that the estimated coefficients were not significant by themselves. They could only be explained in the forms of Allen elasticity of substitution formulas, price elasticities and other economic indicators. Thus, these indices were estimated after estimating the model and performing necessary tests. Estimation of cost function is shown in Table 1.

McElroy R^2 is one criterion determining suitability of the system in estimating iterated nonlinear seemingly unrelated regression system. R^2 was estimated as 0.975, which indicated high explanatory power of the model.

Elasticities Calculation

According to the original model fit results and equations (8) and (9), Allen elasticities of substitution for production factors are presented in Table 2. According to this table, all Allen partial elasticities had the expected signs (negative). In other words, there is an inverse relationship between price and demanded quantity. In addition, labor was a substitution input for capital and other services; moreover, capital inputs and other services were complementary.

According to the substitution relationship between capital and labor, increase in capital costs increases labor employment in the process of cement production. Furthermore, complementary relationship between capital and other services led to the fact that application other services in various stages increased. These services are required for increasing capital utilization.

Table 2 - Results obtained from calculation of Allen partial elasticities between production factors

Variable	Labor	Capital	Other services
Labor	-0.6656	0.9309	0.3657
Capital	-	-0.9588	-0.8192
Other services	-	-	-0.0750

Source: research findings

Cross-price elasticities of production factors were extracted according to equations (10) and (11) in order to estimate composite cost function. The elasticities results are presented in Table 3.

Table 3 - Computational values and cross-price elasticities of production factors at average data level

Variable	Labor	Capital	Other services
Labor	-0.3244	0.4804	0.4416
Capital	0.4537	-0.2323	-0.2213
Other services	0.1782	-0.1985	-0.7834

Source: research findings

According to Table (3), demand price elasticities for all inputs have a negative sign, which confirms the law of demand. In addition, absolute values of all elasticities were also smaller than one, which indicated inelasticity of production factors against changes in factor prices.

Low elasticity of price in terms of labor input shows that the employer cannot react appropriately in case that labor wages rise. In other words, the employer cannot reduce the number of work force as wage increases. This elasticity is less than expected due to labor supporting laws. In addition, the cross-elasticity between labor and capital was equal to 0.4804 while the cross-elasticity between capital and labor was equal to 0.4537. The impact of increasing capital price on demand for labor was higher than the impact of increasing labor price on capital demand. Moreover, the signs of all elasticities for each production factor were compatible with those for Allen - Uzawa partial elasticities.

The results relevant to total and average cost elasticity with respect to mean are presented in Table 4. Since total cost elasticity was less than one, total cost function was inelastic. In other words, the cost in this firm increases with a percentage, which is less than production costs. Therefore, the firm is in the descending position considering average cost; on the other hand, the average cost elasticity with respect to output is negative.

The results relevant to returns to scale based on composite cost function are presented in Table 4. This parameter was calculated as 4.3, which confirmed ascending returns to scale at this firm. Significant relative increase in the production requires smaller relative increase in inputs. As a result, the firm is in downward section of long-term average cost curve. Then, final cost is less than average cost. The results are consistent with those obtained in the previous section.

Table 4 – calculated values of returns to scale, total and average cost elasticity with respect to production

Index	Returns to scale	Average cost elasticity with respect to production	Total cost elasticity with respect to production
Value	4.3277	-0.7689	0.2310

Source: research findings

Technical changes parameter considering average data level was obtained as -0.01819. As a result, there were technological advances in this firm over time. This has led to a reduction in production costs. In other words, technical change in this firm led to the fact that a certain amount of products can be produced with less factors. It can also be concluded that more products can be produced with the same production factors and equipment. This suggests efficiency of the technology used in the firm.

Results

1. Absolute value of total cost elasticity was smaller than one. Then, economies of scale in Fars Cement Company were confirmed. This implied that the firm could reduce the average cost by increasing production scale. Then, the firm can achieve lower levels of average cost.

2. Absolute value of returns to scale was greater than one, which indicated ascending returns to scale at the firm. In other words, if all factors of production increase by a certain ratio, the production will increase more than whatever the production factors had increased.

3. Technical advances parameter reflects the impact of technological changes on the cost function. Based on the results, numerical value of this index was negative,

which suggested a reduction in total costs resulting from technical advances or increased time trend variable. This result showed that as time passes by, the technique used by the factory would improve.

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