Lifetime Employment and a Sequential Choice in a Mixed Duopoly Market with a Joint-Stock Firm

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Abstract

This paper examines a three-stage game model in which a joint-stock private firm and a state-owned public firm can sequentially offer lifetime employment before competing in quantities. The game runs as follows. First, the joint-stock private firm decides whether to offer lifetime employment. Second, the state-owned public firm decides whether to offer lifetime employment. Third, both firms choose their outputs simultaneously and independently. The paper demonstrates that there is an equilibrium solution where only the joint-stock private firm offers lifetime employment.

Keywords: Mixed duopoly, three-stage game, state-owned firm, joint-stock firm, lifetime employment

Introduction

The analysis of mixed oligopoly models including state-owned welfare-maximizing public firms is widely performed by many economists. For example, Mujumdar and Pal (1998) consider a mixed duopoly, with a welfare-maximizing firm and a profit-maximizing firm, producing a homogeneous commodity and find that an increase in tax (ad valorem or specific) does not change total output, but increases the output of the welfare-maximizing firm and the tax revenue. Pal (1998) analyzes the subgame perfect Nash equilibrium of a mixed market, where the firms first choose the timing for selecting their quantities, and finds that the results are strikingly different from those obtained in a

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corresponding oligopoly with all profit maximizing firms. Lu (2007) formulates a mixed oligopoly model in which a single state-owned public firm and foreign private competitors first choose the time period of choosing their output levels, and demonstrates that there is no subgame perfect Nash equilibrium in which all firms produce simultaneously in the same time period. Lu and Poddar (2009) examine a mixed duopoly model of endogenous timing of sequential capacity and output choice, and demonstrate that there exists no subgame perfect Nash equilibrium in which all firms simultaneously play at capacity stage or at the output stage. Barcena-Ruiz and Garzón (2007) examine a two-stage mixed duopoly model in which each firm chooses its capacity and price, and show that if the goods are substitutes, then the private firm chooses over-capacity and the public firm under-capacity. In addition, Ohnishi (2012a) focuses on the role that production subsidies play in a Bertrand mixed duopoly and shows that the results are the same as those of the existing Cournot mixed market literature.

There are also many other excellent studies (see, e.g., Nett, 1994; Willner, 1994; Fjell and Pal, 1996; George and La Manna, 1996; White, 1996; Pal and White, 1998; Poyago-Theotoky, 1998; Wen and Sasaki, 2001; Matsumura, 2003; Beladi and Chao, 2006; Chao and Yu, 2006; Lu and Poddar, 2007; Ohnishi, 2008a; Saha and Sensarma, 2008; Artz, Heywood and McGinty, 2009; Roy chowdhury, 2009; Wang and Wang, 2009; Heywood and Ye, 2010; Wang and Lee, 2010; Pal and Saha, 2014; Cracau, 2015). However, these studies consider mixed oligopoly models in which state-owned firms coexist with profit-maximizing capitalist firms.

Only a few studies consider joint-stock firms. For example, Meade (1972) shows the differences in incentives, short-run adjustment, and so forth among profit-maximizing, labor-managed and joint-stock firms. Hey (1981) restricts attention to the case of a perfectly competitive firm producing a single output with two inputs, labor and capital, and examines the behavior of profit-maximizing, labor-managed and joint-stock firms. Ohnishi (2010) shows the equilibrium outcome of two-stage Cournot duopoly competition with a profit-maximizing firm and a joint-stock firm and finds that the introduction of lifetime employment into the analysis of Cournot mixed competition is profitable only for the joint-stock firm. In addition, Ohnishi (2015) investigate a three-stage mixed duopoly model, where a state-owned public firm and a joint-stock firm are allowed to provide lifetime employment as a strategic device, and concludes that introducing lifetime employment into the model of three-stage mixed duopoly is beneficial for the state-owned firm.

We develop a theory of duopolistic competition between a joint-stock private firm and a state-owned public firm. The game runs as follows. In stage 1, the joint-stock private firm decides whether to offer lifetime employment. In stage 2, the state-owned public firm decides whether to offer lifetime employment. In stage 3, both firms simultaneously and independently choose actual outputs. We analyze the equilibrium outcomes of the three-stage game.

The purpose of this study is to present the equilibrium solution of three-stage mixed duopoly model where a state-owned firm and a joint-stock firm are allowed to offer lifetime employment.
The Model

There is a market composed of one joint-stock profit-per-capital-maximizing firm (firm J) and one state-owned welfare-maximizing firm (firm S). The duopolists produce perfectly substitutable goods. In the balance of this paper, subscripts J and S denote firm J and firm S, respectively. In addition, when \( i \) and \( j \) are used to refer to firms in an expression, they should be understood to denote J and S with \( i \neq j \). We do not consider the possibility of entry or exit. The inverse demand function is represented by \( P = a - Q \), where \( Q = q_J + q_S \) and \( a > Q \).

The timing of the game is as follows. In stage 1, firm J decides whether to offer lifetime employment or not. Firm S observes the behavior of firm J. If firm \( i \) offers lifetime employment, then it chooses an output level \( q_i^* > 0 \) and enters into a lifetime employment contract with the number of employees necessary to achieve \( q_i^* \). In stage 2, firm S decides whether to offer lifetime employment or not. Firm J observes the behavior of firm S. In stage 3, both firms simultaneously and independently choose actual outputs \( q_J \geq 0 \) and \( q_S \geq 0 \).

Therefore, social welfare, which is the sum of consumer surplus and profits, is given by

\[
W = \begin{cases} 
\frac{Q^2}{2} + Pq_S - w_Sq_S^2 - r_Sq_S^2 - f_S + Pq_J - w_Jq_J^2 - r_Jq_J^2 - f_J & \text{if } q_S > q_S^*, \\
\frac{Q^2}{2} + Pq_S - w_Sq_S^2 - r_Sq_S^2 - f_S + Pq_J - w_Jq_J^2 - r_Jq_J^2 - f_J & \text{if } q_S \leq q_S^*,
\end{cases}
\]

(1)

where \( Q^2/2 \) denotes consumer surplus, \( w > 0 \) is the wage rate, \( r > 0 \) is the capital cost for each unit of output, and \( f > 0 \) is the fixed cost.

Firm J’s profit per capital is given by

\[
v_J = \begin{cases} 
Pq_J - w_Jq_J^2 - r_Jq_J^2 - f_J & \text{if } q_J > q_J^*, \\
Pq_J - w_Jq_J^2 - r_Jq_J^2 - f_J & \text{if } q_J \leq q_J^*,
\end{cases}
\]

(2)

where \( k_J > 0 \) denotes the capital inputs. Unlike Ohnishi (2016), we assume that \( k_J \) is a function of \( q_J \). We consider the following production function:

\[ q_J = \sqrt{k_J} \]
From (2) and (3), we can write the objective function of firm J as

\[
v_j = \begin{cases} 
  \frac{Pq_j - w_jq_j^2 - r_jq_j^2 - f_j}{q_j^2} & \text{if } q_j > q_j^*, \\
  \frac{Pq_j - w_jq_j^2 - r_jq_j^2 - f_j}{q_j^2} & \text{if } q_j \leq q_j^*. 
\end{cases}
\]

(4)

If firm \( i \) offers lifetime employment, then the cost of \( w_iq_i^2 \) is sunk. This irreversible behavior by firm \( i \) is communicated to firm \( j \) and causes changes to the quantity-setting competing environment. Firm S aims to maximize social welfare, while firm J aims to maximize its profit per capital. In this paper, we adopt subgame perfection as our solution concept.

**Supplementary Explanations**

In this section, we give supplementary explanations of the model described in the previous section. Firstly, we derive the following reaction functions from (1) and (4):

\[
R_S = \begin{cases} 
  \frac{a - q_j}{1 + 2(w_S + r_S)} & \text{if } q_S > q_S^*, \\
  \frac{a - q_j}{1 + 2r_S} & \text{if } q_S = q_S^*, \\
  \frac{a - q_j}{1 + 2r_S} & \text{if } q_S < q_S^*. 
\end{cases}
\]

(5)

\[
R_j = \begin{cases} 
  \frac{2f_j}{a - q_j} & \text{if } q_j > q_j^*, \\
  \frac{a - q_j}{q_j^*} & \text{if } q_j = q_j^*, \\
  \frac{2(w_jq_j^2 + f_j)}{a - q_S} & \text{if } q_j < q_j^*. 
\end{cases}
\]

(6)

Firm S’s reaction functions slope downward, while firm J’s reaction functions are upward sloping.

Both firms’ reaction curves are displayed in Figure 1. \( R_i^N \) is the reaction curve representing the best quantity choice of firm \( i \) in the response to the quantity sold by firm \( j \), if lifetime employment has not yet been offered. \( R_i^L \) is the reaction curve of firm \( i \), if lifetime employment has already been offered. If firm S selects \( q_S^* \) and offers lifetime employment, then its reaction curve becomes the kinked bold broken line. In addition, if firm J selects \( q_j^* \) and offers lifetime employment, then its reaction curve becomes the kinked bold line.
Secondly, we prove the following two lemmas.

**Lemma 1:** Firm \( i \)'s optimal output is higher when it offers lifetime employment than when it does not.

Proof: First, we prove that firm S’s welfare-maximizing output is larger when it offers lifetime employment than when it does not. From (1), we see that the offer of lifetime employment by firm S will never increase its marginal cost of production. When firm S does not offer lifetime employment, its first-order condition is

\[
a - q_i - (1 + 2w_S + 2r_5)q_S = 0,
\]

and when firm S offers lifetime employment, its first-order condition is

\[
a - q_i - (1 + 2r_5)q_S = 0,
\]
where \( w_s \) is positive. To satisfy (8), \( a - q_j - (1 + 2w_s + 2r_s)q_s \) is negative. Thus, firm S’s optimum output is larger when it offers lifetime employment than when it does not.

Next, we prove that firm J’s profit-per-capital-maximizing output is larger when it offers lifetime employment than when it does not. From (4), we see that the offer of lifetime employment by firm J will never increase its marginal cost of production. When firm J does not offer lifetime employment, its first-order condition is

\[
-aq_j + q_s q_j + 2f_j = 0,
\]

and when firm J offers lifetime employment, its first-order condition is

\[
-aq_j + q_s q_j + 2w_j q_j^* + 2f_j = 0,
\]

where both \( w_j \) and \( q_j^* \) are positive. To satisfy (10), \(-aq_j + q_s q_j + 2f_j \) has to be negative. Thus, Lemma 1 is proved. Q.E.D.

**Lemma 2:** If firm \( i \) offers lifetime employment and an equilibrium is achieved, then at equilibrium \( q_i = q_i^* \).

Proof: First, we prove that if firm S offers lifetime employment, then at equilibrium \( q_s = q_s^* \). Consider the possibility that \( q_s < q_s^* \) at equilibrium. From (1), when firm S offers lifetime employment, social welfare is

\[
W = \frac{Q^2}{2} + P q_s - w_s q_s^2 - r_s q_s^2 - f_s + P q_j - w_j q_j^2 - r_j q_j^2 - f_j
\]

\[
= \frac{Q^2}{2} + P q_s - (w_s + r_s) q_s^2 + w_s \left( q_s^2 - q_j^2 \right) - f_s + P q_j - w_j q_j^2 - r_j q_j^2 - f_j.
\]

Here, if \( q_s < q_s^* \), then firm S has to employ the extra workers. Therefore, firm S can increase social welfare by reducing \( q_s^* \), and the equilibrium solution does not change in \( q_s \leq q_s^* \). Hence, \( q_s < q_s^* \) does not result in an equilibrium.

Consider the possibility that \( q_s > q_s^* \) at equilibrium. From (1), we see that firm S’s cost function is \( w_s q_s^2 + r_s q_s^2 + f_s \). It is impossible for firm S to change its output in equilibrium because such a strategy is not credible. Thus, if \( q_s > q_s^* \), lifetime employment does not function as a strategic commitment device.

Next, we prove that if firm J offers lifetime employment, then at equilibrium \( q_j = q_j^* \). Consider the possibility that \( q_j < q_j^* \) at equilibrium. From (4), when firm J offers lifetime
employment, its profit-per-capital is

\[ v_j = \frac{Pq_j - w_j q_j^2 + r_j q_j^3 - f_j}{q_j^2} = \frac{Pq_j - (w_j + r_j)q_j^2 + w_j \left(q_j^2 - q_j^3\right) - f_j}{q_j^2}. \]

Here, if \( q_i < q_i^* \), then firm J has to employ the extra workers. Therefore, firm J can increase its profit per capital by reducing \( q_i^* \), and the equilibrium solution does not change in \( q_i \leq q_i^* \). Hence, \( q_i < q_i^* \) does not result in an equilibrium.

Consider the possibility that \( q_i > q_i^* \) at equilibrium. From (6), we see that firm J’s cost function is \( w_j q_j^2 + r_j q_j^3 + f_j \). It is impossible for firm J to change its output in equilibrium because such a strategy is not credible. Thus, if \( q_i > q_i^* \), capacity investment does not function as a strategic commitment device. \textit{Q.E.D.}

These lemmas provide characterizations of lifetime employment as a strategic commitment device. Lemma 1 indicates that if firm \( i \) offers lifetime employment, then its optimal output increases. If firm \( i \) offers lifetime employment, the cost of \( w_i q_i^* \) is sunk. Therefore, if \( q_i < q_i^* \), since firm S employs the extra employees, firm \( i \) has to bear the extra cost of \( w_i (q_i^* - q_i) \), and thereby social welfare falls. Lemma 2 means that at equilibrium firm \( i \) does not employ the extra employees.

Thirdly, we consider firm S’s Stackelberg leader output. Firm S selects \( q_s \), and firm J selects \( q_j \) after observing \( q_s \). If firm S is the Stackelberg leader, then it maximizes social welfare \( W(q_s, R_j(q_s)) \) with respect to \( q_s \).

**Lemma 3:** Firm \( i \)’s Stackelberg leader output is higher than its Cournot output.

Proof: First, we consider firm S’s Stackelberg leader output. Firm S selects \( q_s \), and firm J selects \( q_j \) after observing \( q_s \). That is, firm S maximizes social welfare \( W(q_s, R_j(q_s)) \) with respect to \( q_s \). Therefore, firm S’s Stackelberg leader output satisfies the first-order condition:

\[ \frac{\partial W}{\partial q_s} + \frac{\partial W}{\partial q_j} \frac{\partial R_j}{\partial q_s} = 0. \]  
\( (11) \)

Here, \( \frac{\partial W}{\partial q_j} \) is positive. \( \frac{\partial R_j}{\partial q_s} \) is also positive from (6). To satisfy (11), \( \frac{\partial W}{\partial q_s} \) has to be negative. Thus, firm S’s Stackelberg leader output exceeds its Cournot output.
Next, we consider firm J’s Stackelberg leader output. Firm J selects $q_J$, and firm S selects $q_S$ after observing $q_J$. That is, firm J maximizes its profit per capital $V_J(q_J, R_S(q_J))$ with respect to $q_J$. Therefore, firm J’s Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial V_J}{\partial q_J} + \frac{\partial V_J}{\partial q_S} \frac{\partial R_S}{\partial q_J} = 0.$$ 

(12)

Here, $\frac{\partial V_J}{\partial q_J} = -q_J$ is negative. $\frac{\partial R_S}{\partial q_J}$ is also negative from (5). To satisfy (12), $\frac{\partial V_J}{\partial q_J}$ has to be negative. Thus, firm J’s Stackelberg leader output exceeds its Cournot output. Q.E.D.

Lemma 3 indicates that firm $i$ has an incentive to increase its output.

**Equilibrium**

In this section, we discuss the equilibrium of the three-stage game. In this game, first firm J moves, then firm S observes firm J’s move, and subsequently firm S moves. The solution can be stated as follows.

**Proposition 1:** In the three-stage game with firm J moving first and firm S moving second, there exists an equilibrium where only firm J offers lifetime employment.

Proof: First, we prove (i). In stage 1, firm J can offer lifetime employment. Lemma 3 states that firm J’s Stackelberg leader output is higher than its Cournot output without lifetime employment. Furthermore, $V_J = \left( P_J q_J - w_I q_I^2 - r_J q_J^2 - f_J \right) / q_J^2$ is continuous and concave. $R_S(q_J)$ gives firm S’s optimal output for each output of firm J. In $R_S$, $V_J$ is highest at firm J’s Stackelberg leader point, and the further a point on $R_S$ gets from firm J’s Stackelberg leader point, the more $V_J$ decreases. Firm J chooses $q_J^*$ higher than its Cournot output without lifetime employment and offers lifetime employment in stage 1. Lemma 2 states that if firm J offers lifetime employment, then at equilibrium $q_J^*$. Thus, at equilibrium, firm J’s profit per capital is higher than in the Cournot game without lifetime employment.

In stage 2, firm S can offer lifetime employment. From (5), we see that if firm J offer lifetime employment, then its reaction function will have a flat segment at $q_J^*$ level. $W = Q_J^2 / 2 + P_J q_S - w_I q_I^2 - r_S q_S^2 - f_S + P_J q_J - w_I q_I^2 - r_J q_J^2 - f_J$ is continuous and concave. A little change in firm S’s output does not change firm J’s output and decreases social welfare. Therefore, the offer of lifetime employment by firm S decreases social welfare. Our equilibrium concept is subgame perfection, and all information in the model is common knowledge. Therefore, firm J can always influence firm S to offer lifetime
employment by choosing the appropriate level of \( q^*_J \). Thus, firm S does not offer lifetime employment in stage 2. \( Q.E.D. \)

Proposition 1 indicates that lifetime employment is an effective strategy for the joint-stock firm. We use Figure 1 to explain the intuition behind Proposition 1.

In stage 1, firm J is allowed to offer lifetime employment. By strategic choice of lifetime employment, firm J’s best response becomes (6). The offer of lifetime employment by firm J thus creates kinks in the reaction curve at the level of \( q^*_J \). Therefore, if firm J chooses \( q^*_J \) and offers lifetime employment, then its best response curve shifts up for \( q_1 < q^*_J \) and becomes the bold line. The shift size of firm J’s reaction curve is decided by the value of \( w_J \).

In stage 2, firm S is allowed to offer lifetime employment. By strategic choice of lifetime employment, firm S’s best response becomes (5). The offer of lifetime employment by firm S thus creates kinks in the reaction curve at the level of \( q^*_S \). Therefore, if firm S chooses \( q^*_S \) and offers lifetime employment, then its reaction curve shifts right for \( q_S < q^*_S \) and becomes the bold broken line. The shift size of firm S’s reaction curve is decided by the value of \( w_S \).

In stage 3, each firm noncooperatively chooses its actual output. The equilibrium is decided in a Cournot fashion. Hence, if neither firm offers lifetime employment, then the equilibrium occurs at \( C \).

If only firm J chooses \( q^*_J \) and offers lifetime employment, then the reaction curves cross at \( A \). If firm J chooses \( q^*_1 \) in stage 1 and firm S chooses \( q^*_S \) in stage 2, then the reaction curves cross at \( B \). The reaction curve of firm J will have a flat segment at \( q^*_1 \). Social welfare is lower at \( B \) than at \( A \). If firm S offers lifetime employment, then social welfare decreases. Hence, if firm J offers lifetime employment, then firm S has no incentive to do so. Hence, each firm chooses \( q^*_A \) corresponding to \( A \) in stage 3 and the equilibrium occurs at \( A \).

**Conclusion**

We have studied the equilibrium outcome of three-stage competition in which a joint-stock firm and a state-owned firm can sequentially offer lifetime employment before competing in quantities. As a result of this study, we have demonstrated that there is an equilibrium solution where only the joint-stock firm offers lifetime employment.

We have considered a three-stage game. However, in the real world, most firms are faced with long-term competition. In the near future, we will study various long-term
game models consisting of joint-stock and state-owned firms.

References


