

# Capacity and Reaction Functions of Joint-Stock Firms

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## Abstract

This paper investigates two-stage competition with two joint-stock firms. In the first stage, each firm independently chooses a level of capacity. In the second stage, each firm independently chooses an output. The paper shows that each firm's reaction function has a kink at its installed capacity level. In addition, the paper considers interactions between a joint-stock incumbent and a potential joint-stock entrant as well as a pair of established joint-stock firms. The paper demonstrates that there exist cases in which the joint-stock incumbent can deter entry by the potential joint-stock entrant.

**Keywords:** Capacity, joint-stock firm, reaction function, entry.

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## Introduction

The possibility of firms using excess capacity as a strategic device in duopolistic competition has been studied by many economists. For instance, Dixit (1980) demonstrates that an incumbent installing excess capacity in stage one is able to deter entry by a potential entrant in stage two. Ware (1984) considers three-stage competition in which an incumbent installs capacity in stage one, an entrant installs capacity in stage two, and a Cournot duopoly equilibrium is achieved in stage three. He concludes that although his three-stage equilibrium is qualitatively similar to Dixit's two-stage equilibrium, it differs in that the strategic advantage available to the first mover is lessened. In addition, Poddar (2003) examines a two-stage model of strategic entry deterrence (*a la* Dixit 1980) under demand uncertainty, and demonstrates that to improve its strategic position in the product market competition an incumbent will choose a level of capacity that may remain idle in a low state of demand. However, these studies examine the behaviours of profit-maximizing firms.

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We examine the behaviours of joint-stock firms. Mead (1972) shows the differences in incentives, short-run adjustment and so forth among joint-stock, labour-managed and profit-maximizing firms. Hey (1981) restricts attention to the case of a perfectly competitive firm producing a homogeneous final product with inputs of labour and capital, and investigates the behaviours of joint-stock, labour-managed and profit-maximizing firms. In addition, Ohnishi (2010) presents the equilibrium outcome of a quantity-setting model comprising a joint-stock firm and a profit-maximizing firm, demonstrates that introducing lifetime employment into the model of Cournot duopoly competition is beneficial only for the joint-stock firm.

In this paper, we first consider a two-stage model with two joint-stock firms. In the first stage, each firm non-cooperatively chooses a level of capacity. In the second stage, each firm non-cooperatively chooses an output. We show that each firm's reaction function is downward sloping and has a kink at its installed capacity level.

Next, we consider interactions between a joint-stock incumbent and a potential joint-stock entrant rather than a pair of established joint-stock firms. We demonstrate that there are cases in which the joint-stock incumbent can deter entry by the potential joint-stock entrant.

The main purpose of this paper is to show the reaction functions of the joint-stock firms in the duopoly model with capacity.

## The Model

Let us consider a market with two joint-stock profit-per-capacity-maximizing firms, firm 1 and firm 2. We do not consider the possibility of entry or exit. In the remainder of this paper, subscripts 1 and 2 denote firm 1 and firm 2, respectively. In addition, when  $i$  and  $j$  are used, they should be understood to denote 1 and 2 with  $i \neq j$ . The market price is determined by the inverse demand function  $p(Q)$ , where  $Q = q_1 + q_2$ . We assume that  $p' < 0$  and  $p'' \leq 0$ .

The two stages of the game run as follows. In the first stage, each firm simultaneously and independently chooses a level of capacity  $k_i > 0$ . Neither firm can reduce or dispose of capacity. In the second stage, each firm simultaneously and independently chooses and sells an actual output  $q_i > 0$ .

Therefore, firm  $i$ 's profit per capacity is given by

$$\psi_i = \begin{cases} \frac{p(Q)q_i - v_i q_i - f_i}{k_i} & \text{if } q_i > k_i \\ \frac{p(Q)q_i - (v_i - r_i)q_i + r_i k_i - f_i}{k_i} & \text{if } q_i \leq k_i \end{cases} \quad (1)$$

where  $v > 0$  denotes the total cost of each unit of output,  $r \in (0, v)$  is the constant cost per unit of capacity, and  $f > 0$  is the fixed cost. Ohnishi (2015) adopts the capital input function  $k(q_i)$  as a function of output  $q_i$ . Throughout this paper, we use subgame perfection as an equilibrium concept. The fact that inverse demand is defined only for non-negative outputs ensures that all outputs obtained in equilibrium are non-negative.

### Reaction Functions

We state both firms' reaction functions in quantities. We derive firm  $i$ 's best reaction function from (1). If  $q_i > k_i$ , then its reaction function is defined by

$$R_i^v(q_j) = \arg \max_{q_i} \left[ \frac{p(Q)q_i - v_i q_i - f_i}{k_i} \right] \quad (2)$$

and if  $q_i \leq k_i$ , then its reaction function is defined by

$$R_i^r(q_j) = \arg \max_{q_i} \left[ \frac{p(Q)q_i - (v_i - r_i)q_i + r_i k_i - f_i}{k_i} \right] \quad (3)$$

Therefore, if firm  $i$  installs  $k_i$ , then its best response is shown as follows:

$$R_i(q_j) = \begin{cases} R_i^v(q_j) & \text{if } q_i > k_i \\ k_i & \text{if } q_i = k_i \\ R_i^r(q_j) & \text{if } q_i < k_i \end{cases} \quad (4)$$

The Cournot equilibrium occurs where each firm maximizes its objective with respect to its own output, given its rival's output. The equilibrium must satisfy the following conditions: Firm  $i$  aims to maximize its profit per capital with respect to its own output, given firm  $j$ 's output. The Cournot equilibrium must satisfy the following conditions: If  $q_i > k_i$ , then the first-order condition is

$$p' q_i + p - v_i = 0 \quad (5)$$

and the second-order condition is

$$p'' q_i + 2p' < 0 \quad (6)$$

If firm  $i$  installs  $k_i$  and reduces its marginal cost of production, then the first-order condition is

$$p' q_i + p - v_i + r_i = 0 \quad (7)$$

and the second-order condition is

$$p''q_i + 2p' < 0 \tag{8}$$

Moreover, we have

$$R_i^v(q_j) = R_i^r(q_j) = -\frac{p''q_i + p'}{p''q_i + 2p'} \tag{9}$$

Since  $p' < 0$  and  $p'' \leq 0$ ,  $p''q_i + p'$  is negative. We now state the following proposition:

**Proposition 1:** Under Cournot competition, both  $R_i^v(q_j)$  and  $R_i^r(q_j)$  are downward sloping.

Both firms' reaction curves are illustrated in figure 1.  $R_i^v$  is the reaction curve representing the best quantity choice of firm  $i$  in the response to the quantity sold by firm  $j$ , if it has to incur the full marginal costs of producing any given quantity.  $R_i^r$  is the reaction curve of firm  $i$ , if capacity has been sunk up to an amount at least equal to the desired quantity.

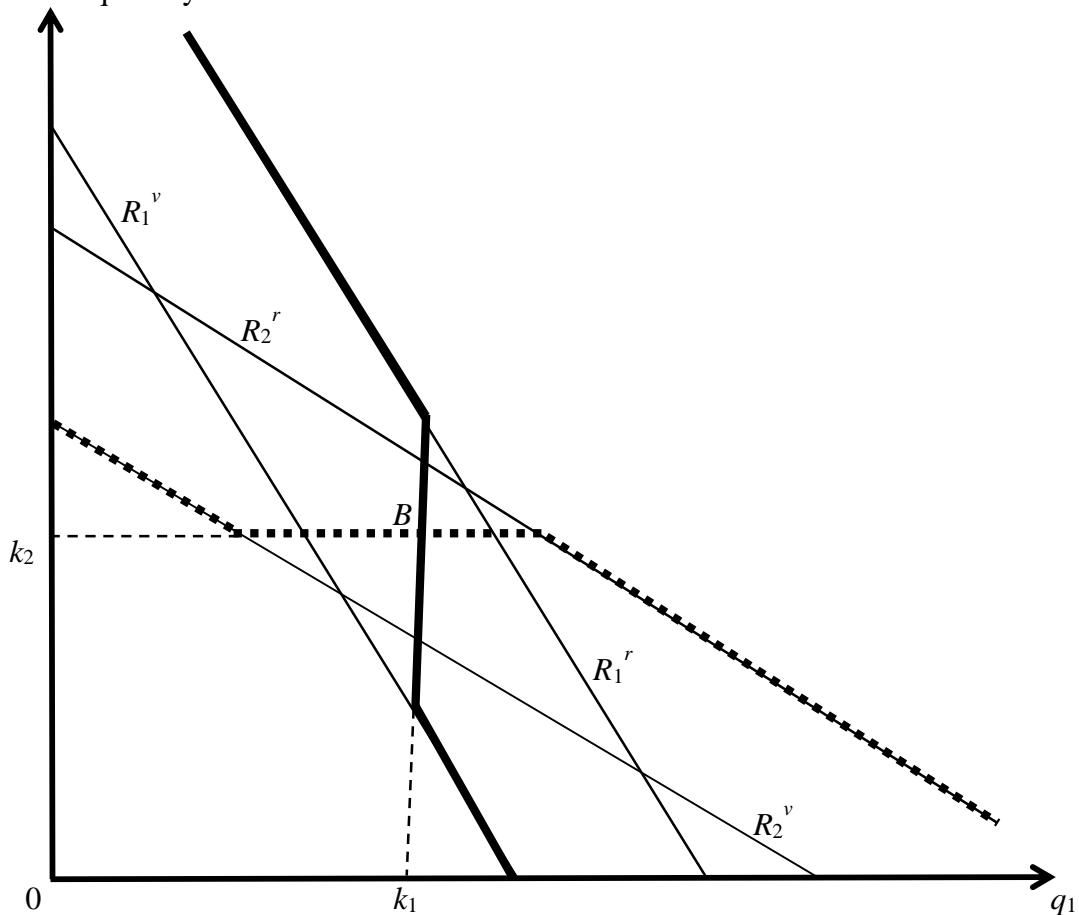


Figure 1 Capacity and reaction curves

## Entry

Suppose that firms 1 and 2 have installed levels of capacity equal to  $k_1$  and  $k_2$ . From (4), firm 1's reaction curve is the kinked bold line, and firm 2's reaction curve is the kinked broken bold line. Each firm's reaction curve will have a flat segment at its installed capacity level.

In this section, the rival is taken to be a potential entrant rather than an established firm. That is, the following situation is considered. In the first stage, firm 1 (established firm) chooses a level of capacity, and firm 2 (potential entrant) observes the behaviour of firm 1. In the second stage, firm 2 decides whether to enter the market. If firm 2 enters, a Cournot duopoly equilibrium is achieved, whereas if firm 2 does not enter, firm 1 keeps its monopoly position in the market.

We suppose that firm 2's reaction curve meets  $R_1^v$  at  $C = (q_1^C, q_2^C)$  and  $R_1^r$  at  $E = (q_1^E, q_2^E)$  as shown in figure 2. Furthermore, we suppose  $D = (q_1^D, q_2^D)$  as a point on firm 2's reaction curve, where its post-entry profit per capacity is zero and it does not enter. Firm 2's reaction curve is discontinuous at  $D$ , made up of the two segments illustrated by the bold line in figure 2.

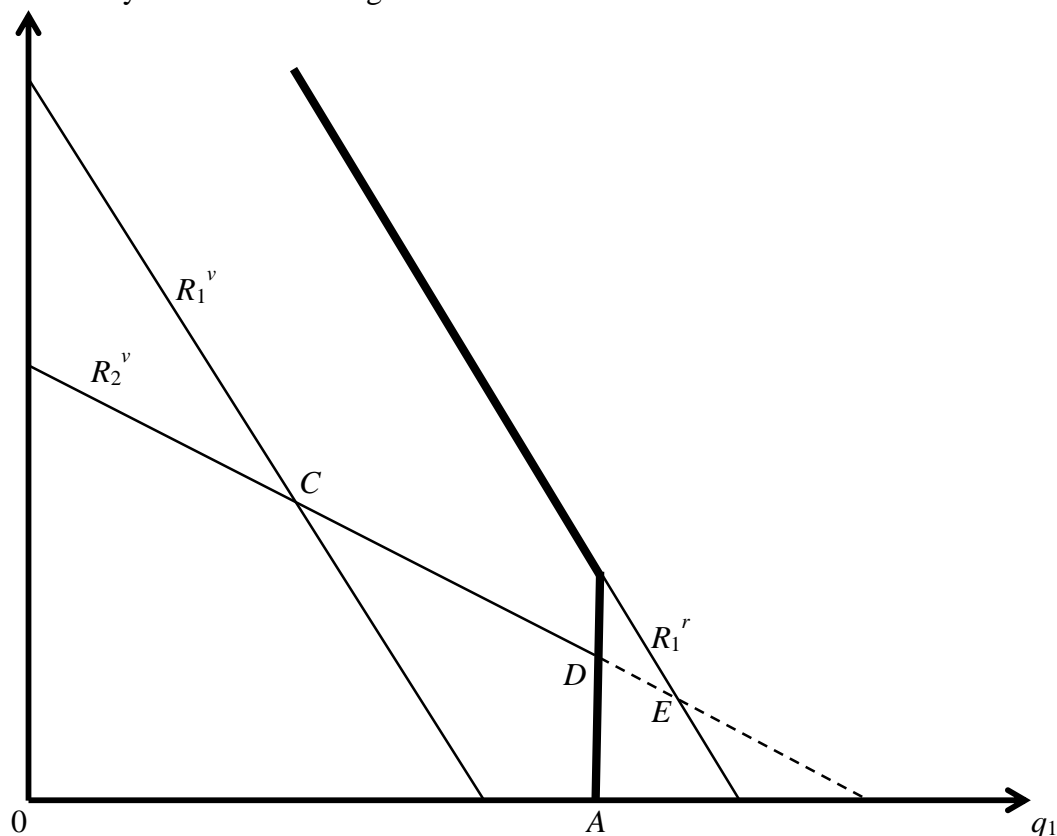


Figure 2 Entry deterrence

There are three cases to consider.

$$(i) q_1^D \leq q_1^C$$

Firm 2 does not try to enter the market at all. Firm 2's entry is blocked. Entry being irrelevant, firm 1 will enjoy a pure monopoly.

$$(ii) q_1^C < q_1^D \leq q_1^E$$

Firm 2 is either accommodated into the market or deterred. In accommodated, the post-entry equilibrium occurs at the most appropriate point for firm 1 between  $C$  and  $D$  on firm 2's reaction curve. In deterred, firm 1 installs  $k_1^D$  corresponding to  $D$ . Hence, if  $q_1^C < q_1^D \leq q_1^E$ , then the entry-deterrence equilibrium occurs at  $A$ .

$$(iii) q_1^E < q_1^D$$

Firm 1 cannot hope to prevent entry, so it can only seek the best available duopoly position. Depending on firm 1's choice of  $k_1$ , the post-entry equilibrium can be at any point between  $C$  and  $E$  on firm 2's reaction curve. The point  $C$  corresponds to firm 1's smallest output that can be sustained as a Cournot equilibrium. The point  $E$  is firm 1's largest output that can be sustained as a Cournot equilibrium. Therefore, the post-entry equilibrium occurs at the most appropriate point for firm 1 between  $C$  and  $E$ .

## Conclusion

We have examined duopoly competition in which each joint-stock firm chooses a level of capacity. We have established that the joint-stock firm's reaction function is downward sloping and has a kink at the level of capacity. Next we have considered interactions between a joint-stock incumbent and a potential joint-stock entrant rather than a pair of established joint-stock firms. The joint-stock incumbent does not always deter entry by choosing excess capacity, but we have demonstrated that it is a possibility.

In this paper, we have examined a two-stage model. In the future, we will study various long-run markets where joint-stock firms exist.

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