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Reaction Functions under Quantity Competition in International Mixed Triopolies

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Abstract

This paper considers an economy in which a state-owned public firm, a domestic private firm and a foreign private firm compete in output levels. The paper examines the firms' reaction functions in the economy. As a result of this analysis, the paper shows that there is a mixed triopoly in which the state-owned public firm acts as a strategic complementor to the domestic private firm's output, that is, its reaction function is upward sloping.

Keywords: Reaction functions, Quantity competition, State-owned public firm, Domestic private firm, Foreign private firm.

Introduction

It is very well known that in oligopoly competition, firms' Cournot reaction functions are downward sloping. However, Delbono and Scarpa (1995) examine a mixed duopoly model in which a state-owned public firm competes with a private firm, and show that, under the strict concavity of the demand function, the public firm's reaction function may be upward sloping in the space of output levels whenever the public firm maximizes social welfare but gives less weight to the private firm's profit than to its own profit. In addition, Matsumura (2003) considers an international mixed duopoly model in which a domestic public firm competes against a foreign private firm, and demonstrates that the public firm's reaction function is upward sloping in the space of output levels.

State-owned public firms exist in developed and developing countries as well as in former communist countries. Competition between public and private firms can be observed in many industries such as broadcasting, education, electricity,

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telecommunications, tobacco, banking, home loans, life insurance, health care and shipbuilding.

We examine a market in which a state-owned public firm, a domestic private firm and a foreign private firm compete in output levels. The purpose of this study is to present the firms' reaction functions in the market. As a result of this analysis, we show that the state-owned public firm may act as a strategic complementor to the domestic private firm's output as well as to the foreign private firm's output.

The remainder of this paper is organized as follows. In the next section, we describe the basic model. The third section presents the result of this study. The final section contains a brief conclusion.

Basic Setup

There is an economy composed of one foreign private firm (firm F), one domestic private firm (firm D) and one state-owned public firm (firm S). Throughout this paper, subscripts F, D and S denote firm F, firm D and firm S, respectively. In addition, when i, j and k are used to refer to firms in an expression, they should be understood to represent F, D and S with $i \neq j \neq k$. There is no possibility of entry or exit.

The triopolists produce perfectly substitutable goods. The inverse demand function is represented by $P(Q)$, where $Q = q_F + q_D + q_S$ denotes the total quantity demanded in the market. We assume that $dP / dQ = (\partial P / \partial q_F = \partial P / \partial q_D = \partial P / \partial q_S) < 0$ and $d^2P / dQ^2 = (\partial^2 P / \partial q_F^2 = \partial^2 P / \partial q_D^2 = \partial^2 P / \partial q_S^2) < 0$. Note that the demand function is strictly concave. We consider a static noncooperative game with complete information.

Each firm i's profit is given by

$$\pi_i = P(Q)q_i - c_i(q_i) \tag{1}$$

where c_i denotes firm i's cost function. Firm F and firm D choose q_F and q_D, respectively, to maximize their own profits.

Domestic economic welfare, which is defined as the sum of consumer surplus and total profits of the domestic firms, is given by

$$W = \int_0^Q P(x)dx - c_S(q_S) - c_D(q_D) - Pq_F \tag{2}$$

Firm S chooses q_S in order to maximize (2).

We assume that firms have identical technologies with $dc_i / dq_i > 0$ and $d^2c_i / dq_i^2 > 0$. This assumption is often used in literature studying mixed oligopoly markets. If $d^2c_i / dq_i^2 \leq 0$, then firm S maximizes domestic economic welfare by supplying monopolistically in the market. Many works on mixed oligopoly such as Delbono and Scarpa (1995), Fjell and Heywood (2002), Bárcena-Ruiz and Garzón (2003), Matsumura and Kanda (2005),

Kato (2008), Ohnishi (2008, 2016), Roy Chowdhury (2009), and Wang and Wang (2009) also eliminate such a trivial solution.

Reaction Functions

In this section, we discuss each firm i's best response $R_i(q_j, q_k)$. First, we derive firm F's reaction functions in quantities from (1). Firm F seeks to maximize its profit with respect to q_F, given q_D and q_S. Therefore, the first-order condition for profit maximization is

$$q_F \frac{dP}{dQ} + P - \frac{dc_F}{dq_F} = 0 \tag{3}$$

and the second-order condition for profit maximization is

$$q_F \frac{d^2P}{dQ^2} + 2\frac{dP}{dQ} - \frac{d^2c_F}{dq_F^2} < 0 \tag{4}$$

Moreover, we obtain

$$\frac{\partial R_F(q_D, q_S)}{\partial q_D} = \frac{\partial R_F(q_D, q_S)}{\partial q_S} = -\frac{dP/dQ + q_F\, d^2P/dQ^2}{2\, dP/dQ + q_F\, d^2P/dQ^2 - d^2c_F/dq_F^2} \tag{5}$$

Equation (5) tells us how firm F will react to the rival's choice of output. We now state the following proposition.

Proposition 1: Both $\partial R_F(q_D, q_S)/\partial q_D$ and $\partial R_F(q_D, q_S)/\partial q_S$ are negative, and thus firm F's reaction functions are downward sloping.

Proposition 1 means that firm F acts as a strategic substitutor to both q_D and q_S. This result is well known. Since firm D's reaction functions are essentially the same as firm F's, their discussions are omitted.

Secondly, we consider firm S's reaction function to firm F's action. Firm S seeks to maximize domestic economic welfare with respect to q_S, given q_F and q_D. Therefore, the first-order condition for welfare maximization is

$$P - \frac{dc_S}{dq_S} - q_F \frac{dP}{dQ} = 0 \tag{6}$$

and the second-order condition for welfare maximization is

$$\frac{dP}{dQ} - \frac{d^2c_S}{dq_S^2} - q_F \frac{d^2P}{dQ^2} < 0 \tag{7}$$

In addition, we obtain

$$\frac{\partial R_S(q_F, q_D)}{\partial q_F} = \frac{q_F\, d^2P/dQ^2}{dP/dQ - d^2c_S/dq_S^2 - q_F\, d^2P/dQ^2} \tag{8}$$

We can now present the following proposition.

Proposition 2: $\partial R_S(q_F, q_D) / \partial q_F$ is positive, and thus firm S's reaction function to firm F's action is upward sloping.

Proposition 2 indicates that firm S acts as a strategic complementor to q_F. This is shown in Delbono and Scarpa (1995) and Matsumura (2003).

Thirdly, we consider firm S's reaction function to firm D's action. Firm S maximizes domestic economic welfare with respect to q_S, given q_F and q_D. Therefore, the first-order condition for welfare maximization is (6) and the second-order condition is (7). Moreover, we have

$$\frac{\partial R_S(q_F, q_D)}{\partial q_D} = -\frac{dP/dQ - q_F\, d^2P/dQ^2}{dP/dQ - d^2c_S/dq_S^2 - q_F\, d^2P/dQ^2} \tag{9}$$

Equation (9) tells us how firm S will react to firm D's choice of output. The main result of this study is stated in the following proposition.

Proposition 3: Under quantity competition,

(i) if $dP/dQ - q_F\, d^2P/dQ^2 < 0$, then $\partial R_S(q_F, q_D)/\partial q_D$ is negative, and firm S's reaction function to firm D's output is downward sloping;

(ii) if $0 < dP/dQ - q_F\, d^2P/dQ^2 < d^2c_S/dq_S^2$, then $\partial R_S(q_F, q_D)/\partial q_D$ is positive, and firm S's reaction function to firm D's output is upward sloping.

Note that if $d^2c_S/dq_S^2 < dP/dQ - q_F\, d^2P/dQ^2$, then (7) is not satisfied. Proposition 3 indicates that $\partial R_S(q_F, q_D)/\partial q_D$ slopes downward if q_F is low and upward if q_F is relatively high. In Proposition 3 (ii), we see that firm S acts as a strategic complementor to both q_D and q_F.

Conclusion

We have investigated a market in which a foreign private firm, a domestic private firm and a state-owned firm compete in output levels. As a result of this analysis, we have shown that there is a mixed triopoly in which the state-owned firm's reaction function to the domestic private firm's output is upward sloping.

In this paper, we have considered a single-shot game. In the future, we will extend our model to various long-term situations.

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