

# Endogenous Timing in Mixed Duopoly with Wage-Rise Contracts as Strategic Device

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## Abstract

This paper considers a mixed duopoly market in which a private firm competes against a public firm. Each firm first has to choose the timing for offering a wage-rise contract as a strategic device. The timing of the game is as follows. In stage one, each firm chooses either stage two or stage three simultaneously and independently. In stage two, the firm choosing stage two offers a wage-rise contract in this stage. In stage three, the firm choosing stage three offers a wage-rise contract in this stage. At the end of the game, each firm chooses its actual output simultaneously and independently. The paper studies the behavior of the public firm and the private firm in the mixed duopoly model. The aim of this paper is to present the equilibrium outcome of the mixed duopoly model.

**Keywords:** Endogenous timing, private firm, public firm, wage-rise contract.

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## Introduction

The analysis of Hamilton and Slutsky (1990) considers two different duopoly games in which each private firm selects the timing for choosing its quantity, price or product type. They consider the pure-strategy subgame perfect equilibria of the games and demonstrate that if one reaction function is upward sloping while the other is downward sloping, one firm does better in the solution of the sequential-move game than in that of the simultaneous-move game.

In the present paper, I examine a mixed oligopoly game model where a profit-maximizing private firm competes with a state-owned welfare-maximizing public firm.

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As is very well known, mixed oligopoly markets exist in many developing, advanced, and former communist economies. Competition between private and public firms is widely observed in many industries including tobacco, telecommunications, shipbuilding, rail, life insurance, home loans, health care, electricity, education, broadcasting, banking, and airlines. There are many works based on the first theoretical analysis of Merrill and Schneider (1966) (for instance, see, Nett, 1994; Willner, 1994; Beladi and Chao, 2006; Bárcena-Ruiz, 2007; Lu and Poddar, 2007, 2009; Ohnishi, 2008, 2015; Saha and Sensarma, 2008; Heywood and Ye, 2010; Xu, Lee, and Wang, 2018).

Pal (1998) examines the subgame perfect Nash equilibrium of a sequential-move mixed oligopoly model with perfectly substitutable goods in which one public and  $N \geq 1$  private firms first have to select the timing for choosing their quantity levels. He demonstrates that the results substantially differ from those of a corresponding pure private oligopoly game.

I analyze the behavior of a private firm and a public firm in a mixed duopoly model. Each firm first has to choose the stage for offering the wage-rise-contract policy (henceforward referred to as WRCP).<sup>2</sup> The following timing is considered. In stage one, each firm independently chooses either stage two or stage three. In stage two, the firm choosing stage two adopts WRCP in this stage. In stage three, the firm choosing stage three offers WRCP in this stage. In stage four, each firm non-cooperatively decides its actual output level.

The purpose of this paper is to show the equilibrium outcome of the mixed duopoly model with private and public firms.

### Basic Model

In this section, I formulate a mixed duopoly model with a public firm (firm 1) and a private firm (firm 2). Throughout this paper, when  $i$  and  $j$  are used to denote firms in an expression, they represent 1 and 2 with  $i \neq j$ . The firms produce and sell perfect substitute goods. Market demand is given by the inverse demand function  $P(X)$ , where  $P$  is the price and  $X = \sum_{i=1}^2 x_i$ . It is assumed that  $P' < 0$  and  $P'' \leq 0$ .

Firm  $i$ 's profit function is

$$\pi_i = P(X)x_i - c_i(x_i) \quad (i = 1, 2), \quad (1)$$

where  $c_i(x_i)$  represents the cost function. It is assumed that  $c_i' > 0$  and  $c_i'' > 0$ . This assumption is adopted in many works (for example, see, Ohnishi, 2010; Hsu, Lee, and Wang, 2018; Xu, Lee, and Wang, 2018). If the marginal cost of production is decreasing or constant, firm 1 becomes the monopolist and optimizes social surplus. Firm 2 aims to maximize (1).

Social surplus ( $S$ ) is the sum of total profits and consumer surplus, and is given by

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<sup>2</sup> For details see Ohnishi (2003).

$$S = \int_0^X P(q) dq - c_1(x_1) - c_2(x_2). \quad (2)$$

Firm 1 seeks to maximize (2).

In stage 1, each firm non-cooperatively decides  $t_i \in (2, 3)$ , where  $t_i$  represents when to offer WRCP. That is,  $t_i = 2$  means that firm  $i$  offers WRCP in stage 2, and  $t_i = 3$  means that it offers WRCP in stage 3. At the end of stage 1, firm  $i$  observes  $t_j$ . In stage 2, firm  $i$  selecting  $t_i = 2$  offers WRCP in this stage. In stage 3, firm  $i$  selecting  $t_i = 3$  offers WRCP in this stage. When firm  $i$  adopts WRCP, it selects a wage premium rate  $w_i > 0$  and a quantity level  $x_i^* \geq 0$ , and agrees to uniformly pay a wage premium to each employee if its actual quantity exceeds  $x_i^*$ . In stage 4, each firm non-cooperatively decides its actual quantity  $x_i > 0$ .

Hence, firm  $i$ 's cost changes as follows:

$$c_i^W(x_i, x_i^*, w_i) = \begin{cases} c_i(x_i) & \text{if } x_i \leq x_i^*, \\ c_i(x_i) - (x_i - x_i^*)w_i & \text{if } x_i \geq x_i^*. \end{cases} \quad (3)$$

Furthermore, firm  $i$ 's profit becomes

$$\pi_i = P(X)x_i - c_i^W(x_i, x_i^*, w_i), \quad (4)$$

and social surplus becomes

$$S = \int_0^X P(q) dq - c_1^W(x_1, x_1^*, w_1) - c_2^W(x_2, x_2^*, w_2). \quad (5)$$

Throughout this study, I analyze the subgame perfect Nash equilibrium of this game.

### Best Response Functions

First, firm 1's best response is derived from (2). When firm 1's marginal cost of production is  $c_1'$ , its best response function is

$$R_1(x_2) = \arg \max_{x_1} \left[ \int_0^X P(q) dq - c_1(x_1) - c_2(x_2) \right]. \quad (6)$$

When firm 1's marginal cost of production is  $c_1' + w_1$ , its best response function is

$$R_1^w(x_2) = \arg \max_{x_1} \left[ \int_0^X P(q) dq - c_1(x_1) - c_2(x_2) + (x_1 - x_1^*)w_1 \right]. \quad (7)$$

Therefore, firm 1's reaction function is as follows:

$$R_1^W(x_2) = \begin{cases} R_1(x_2) & \text{if } x_1 < x_1^*, \\ x_1^* & \text{if } x_1 = x_1^*, \\ R_1^w(x_2) & \text{if } x_1 > x_1^*. \end{cases} \quad (8)$$

The solution outcome is where firm  $i$  optimizes objective function value on own quantity, given firm  $j$ 's quantity. Firm 1 seeks to optimize welfare on  $x_1$ , given  $x_2$ . Therefore, the solution outcome has to satisfy the first order condition (henceforth referred to as FOC) for (6) given by

$$P - c_1' = 0, \quad (9)$$

and the FOC for (7) given by

$$P - c_1' + w_1 = 0. \quad (10)$$

In addition, the following equation is obtained:

$$R_1'(x_2) = R_1^{w'}(x_2) = -\frac{P'}{P'}. \quad (11)$$

Now the following lemma can be stated.

**Lemma 1:**  $R_1(x_2)$  and  $R_1^W(x_2)$  both slope downward.

Lemma 1 indicates that firm 1's optimal response to more aggressive play by firm 2 is to be less aggressive. Bulow, Geanakoplos, and Klemperer (1985) call this property strategic substitutes.

Next, firm 2's best response is derived from (1). When firm 2's marginal cost of production is  $c_2'$ , its best response function is defined by

$$R_2(x_1) = \arg \max_{x_2} [P(X)x_2 - c_2(x_2)]. \quad (12)$$

When firm 2's marginal cost of production is  $c_2' + w_2$ , its best response function is

$$R_2^W(x_1) = \arg \max_{x_2} [P(X)x_2 - c_2(x_2) - (x_2 - x_2^*)w_2]. \quad (13)$$

Therefore, firm 2's reaction function is

$$R_2^W(x_1) = \begin{cases} R_2(x_1) & \text{if } x_2 < x_2^*, \\ x_2^* & \text{if } x_2 = x_2^*, \\ R_2^w(x_1) & \text{if } x_2 > x_2^*. \end{cases} \quad (14)$$

Firm 2 seeks to optimize profit on  $x_2$ , given  $x_1$ , and the solution has to satisfy the FOC for (12) given by

$$P'x_2 + P - c'_2 = 0, \quad (15)$$

and the FOC for (13) given by

$$P'x_2 + P - c'_2 - w_2 = 0. \quad (16)$$

In addition, the following equation is obtained:

$$R'_2(x_1) = R_2'^w(x_1) = -\frac{P' + P''x_2}{2P' + P''x_2 - c_2''}. \quad (17)$$

The following lemma is now stated.

**Lemma 2:**  $R_2(x_1)$  and  $R_2^w(x_1)$  both slope downward.

In the next section, the equilibrium of the mixed market model is discussed.

### Equilibrium

In this section, I start with the following two lemmas.

**Lemma 3:** When firm  $i$  offers WRCP,  $x_i^*$  equals its equilibrium output.

Proof: I first prove the case in which firm 1 offers WRCP. I suppose that at equilibrium  $x_1 > x_1^*$ . From (2), social surplus is

$$S^w = \int_0^x P(q) dq - c_1(x_1) - c_2(x_2) + (x_1 - x_1^*)w_1.$$

Here, if  $x_1 > x_1^*$ , firm 1 need to pay  $(x_1 - x_1^*)w_1$  to its employees. It is possible for firm 1 to increase social surplus by reducing  $x_1$ . The equilibrium outcome never changes in  $x_1 \geq x_1^*$ . Therefore,  $x_1 > x_1^*$  never results in an equilibrium.

I suppose that at equilibrium  $x_1 < x_1^*$ . From (2), firm 1's marginal cost of production is  $c_1'$ . Firm 1 is not able to change its equilibrium output because such a strategic behavior is never credible. Therefore, if  $x_1 < x_1^*$ , WRCP never functions as a strategic device.

Next, I prove the case in which firm 2 offers WRCP. I suppose that  $x_2 > x_2^*$  in equilibrium. From (1), firm 2's profit is

$$\pi^w = P(X)x_2 - c_2(x_2) + (x_2 - x_2^*)w_2.$$

If  $x_2 > x_2^*$ , firm 2 must pay  $(x_2 - x_2^*)w_2$  to its employees. It is possible for firm 2 to increase its profit by reducing  $x_2$ . The solution does not change in  $x_2 \geq x_2^*$ . Therefore,  $x_2 > x_2^*$  does not result in an equilibrium.

I suppose that  $x_2 < x_2^*$  at equilibrium. From (1), firm 2's marginal cost of production is  $c_2'$ . Firm 2 is not able to change its equilibrium output because such a strategic behavior

is never credible. Therefore, if  $x_2 < x_2^*$ , WRCP never functions as a strategic device. Q.E.D.

**Lemma 4:** Firm  $i$ 's optimum quantity when adopting WRCP is lower than that when not adopting WRCP.

Proof: I first prove that firm 1's social-surplus-maximizing quantity when it adopts WRCP is lower than that when it does not. From (2), it is seen that WRCP never decreases firm 1's marginal cost of production. If firm 1's marginal cost of production is  $c'_1$ , the FOC is (9), and if its marginal cost of production is  $c'_1 + w_1$ , the FOC is (10). Here, the sign of  $w_1$  is plus. Therefore,  $P - c'_1$  has to be positive so as to satisfy (10). Hence, firm 1's optimal quantity when its marginal cost of production is  $c'_1 + w_1$  is lower than that when its marginal cost of production is  $c'_1$ .

I next prove that firm 2's profit-maximizing quantity when it adopts WRCP is lower than that when it does not. From (1), it is seen that WRCP never decrease firm 2's marginal cost of production. If firm 2's marginal cost of production is  $c'_2$ , the FOC is (15), and if its marginal cost of production is  $c'_2 + w_2$ , the FOC is (16). Here, the sign of  $w_2$  is plus. Therefore,  $P'x_2 + P - c'_2$  has to be positive so as to satisfy (16). Thus, this lemma is proved. Q.E.D.

I now discuss the equilibrium of the model described in Section 2. I first suppose that firm 1 unilaterally offers WRCP. If firm 1 unilaterally adopts WRCP, then its marginal cost of production rises and hence it diminishes its quantity. Given firm 2's quantity, a decrease in firm 1's quantity diminishes the total quantity and brings down the price. Therefore, firm 2's profit rises. In addition, firm 2 raises its quantity supplied because actions are strategic substitutes. Thus, social surplus may rise.

Next, I consider the possibility that firm 2 unilaterally offers WRCP. When only firm 2 adopts WRCP, its marginal cost of production rises and hence it diminishes its quantity. Given firm 1's quantity, a decrease in firm 2's quantity diminishes the total quantity and increases the price. In addition, firm 1 raises its quantity supplied because actions are strategic substitutes. Given firm 2's quantity, an increase in firm 1's quantity increases the total quantity and brings down the price. Therefore, firm 2's profit reduces.

The following proposition states the main result of this paper.

**Proposition 1:** There is an equilibrium solution in which  $t_1 = 2$  and  $t_2 = 3$ . In the equilibrium solution,  $S^E > S^C$  and  $\pi_2^E > \pi_2^C$ , where the superscripts  $E$  and  $C$  denote the equilibrium solution of this game and the equilibrium solution of the quantity-setting model with no WRCP, respectively.

Proof: I begin by proving whether firm 2 will unilaterally adopt WRCP or not. I examine firm 2's leader quantity when marginal cost of production is constantly equal to  $c'_i$ . Firm 2 chooses  $x_2$ , and after observing  $x_2$ , firm 1 chooses  $x_1$ . The Stackelberg leader (firm 2) optimizes  $\pi_2(x_2, R_1(x_2))$  on  $x_2$ . Hence, firm 2's quantity has to satisfy the FOC:

$$P - c'_2 + P'x_2 + P'x_2R'_1 = 0. \quad (18)$$

By  $R'_1 < 0$  and  $P' < 0$ ,  $P - c'_2 + P'x_2$  needs to be negative so as to satisfy (18). Therefore, firm 2's leader quantity exceeds its simultaneous-move quantity.

Lemma 2 proves that firm 2's optimal quantity when firm 2 adopts WRCP is lower than that when it does not. Hence, firm 2's unilateral adoption of WRCP lowers its profit.

I show that firm 1 adopts WRCP. I consider firm 1's leader quantity when marginal cost of production is constantly equal to  $c'_i$ . Firm 1 chooses  $x_1$ , and after observing  $x_1$ , firm 2 chooses  $x_2$ . The Stackelberg leader (firm 1) optimizes  $S(x_1, R_2(x_1))$  on  $x_1$ . Hence, firm 1's quantity has to satisfy the FOC:

$$P - c'_1 - P'x_2R'_2 = 0. \quad (19)$$

By  $R'_2 < 0$  and  $P' < 0$ ,  $P - c'_1$  needs to be plus so as to satisfy (19). Therefore, firm 1's Cournot quantity exceeds its Stackelberg leader quantity.

Lemma 4 proves that welfare-maximizing quantity when firm 1 offers WRCP is lower than that when it does not. By (3) and (4), it is seen that a decrease of firm 1's quantity is determined by  $w_1$  that is able to be any value above 0.

Firm 2 hopes that firm 1 offers WRCP. By  $S^E > S^C$ , firm 1 chooses  $x_1^{*E}$  and  $w_1^E$ , and offers WRCP. It is supposed that also firm 2 offers WRCP. Furthermore, Lemma 1 proves that  $x_1^* = x_1$  at equilibrium. By (8), it is seen that firm 1's response curve produces a slope of zero at  $x_1^* = x_1$ . This means that  $x_1$  is constant even if  $x_2$  is increased. Therefore, firm 2 is able to raise its profit by raising  $x_2^*$  and  $x_2$ . Firm 2 optimizes its profit by increasing  $x_2^*$  and  $x_2$  to a point of  $R_2$ . Thus, firm 2 chooses  $R_1(x_2^*)$  and offers WRCP.

When firm 1 is the leader, it optimizes social surplus and is able to select its Cournot quantity. Therefore, I have  $S^E \geq S^C$ . Let  $S = \int_0^X P(q)dq - c_1(x_1) - c_2(x_2)$  be concave and continuous on  $x_1$ . Social surplus is largest at firm 1's leader solution in  $R_2$ , and the farther the solution on  $R_2$  is from firm 1's leader solution, the lower social surplus is. Firm 1's Cournot quantity exceeds its Stackelberg leader quantity. Lemma 1 proves that in equilibrium  $x_1 = x_1^*$ . Hence,  $S^E > S^C$ .

I show that  $\pi_2^E > \pi_2^C$ . Firm 1's Nash quantity for the simultaneous-move game exceeds its leader quantity. Since  $\partial\pi_2/\partial x_1 = P'x_2 < 0$ , a decrease in  $x_1$  raises  $\pi_2$  given  $x_2$ . The optimum strategy by firm 2 has to have at least this profit. Hence,  $\pi_2^E > \pi_2^C$ . Q.E.D.

## Conclusion

In this paper, I have examined a mixed duopoly game model in which a private firm and a public firm first simultaneously and independently decide the timing for offering WRCP. It has been demonstrated that the public firm adopts WRCP as leader. In consequence, It has been found that WRCP as a strategic commitment device is beneficial for the public and private firms.

## References

- Bárcena-Ruiz, J. C. (2007). Endogenous timing in a mixed duopoly: price competition. *Journal of Economics*, 91, 263-272.
- Beladi, H., & Chao, C.-C. (2006). Does privatization improve the environment? *Economics Letters*, 93, 343-347.
- Bulow, J., Geanakoplos, J., & Klemperer, P. (1985). Multimarket oligopoly: strategic substitutes and complements. *Journal of Political Economy*, 93, 488-511.
- Hamilton, J. H., & Slutsky, S. M. (1990). Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behavior*, 2, 29-46.
- Heywood, J. S., & Ye, G. (2010). Optimal privatization in a mixed duopoly with consistent conjectures. *Journal of Economics*, 101, 231-246.
- Hsu, C.-C. Lee, J.-Y., & Wang, L. F. S. (2018). Undesirable free entry in oligopoly with foreign competitors. In K. Ohnishi, *Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 3* (pp. 127-143). Sharjah, UAE: Bentham Science Publishers.
- Lu, Y., & Poddar, S. (2007). Firm ownership, product differentiation and welfare. *The Manchester School*, 75, 210-217.
- Lu, Y., & Poddar, S. (2009). Endogenous timing in a mixed duopoly and private duopoly – ‘capacity-then-quantity’ game: the linear demand case. *Australian Economic Papers*, 48, 138-150.
- Merrill, W., & Schneider, N. (1966). Government firms in oligopoly industries: a short-run analysis. *Quarterly Journal of Economics*, 80, 400-412.
- Nett, L. (1994). Why private firms are more innovative than public firms. *European Journal of Political Economy*, 10, 639-653.
- Ohnishi, K. (2003). A model of a price-setting duopoly with a wage-rise contract. *Australian Economic Papers*, 42, 149-157.
- Ohnishi, K. (2008). Mixed duopoly with wage-rise contract as strategic commitments. *Ekonomia*, 11, 35-49.
- Ohnishi, K. (2010). Wage-rise contract and endogenous timing in international mixed duopoly. *Research in Economics*, 64, 121-127.
- Ohnishi, K. (2015). Wage-rise contract and a three-stage model with state-owned and labour-managed firms. In K. Ohnishi, *Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 1* (pp. 139-154). Sharjah, UAE: Bentham Science Publishers.



- Pal, D. (1998). Endogenous timing in a mixed oligopoly. *Economics Letters*, 61, 181-185.
- Saha, B., & Sensarma, R. (2008). The distributive role of managerial incentives in a mixed duopoly. *Economics Bulletin*, 12, 1-10.
- Willner, J. (1994). Welfare maximization with endogenous average costs. *International Journal of Industrial Organization*, 12, 373-386.
- Xu, L., Lee, S.-H., & Wang, L. F. S. (2018). Strategic trade and privatization policies in bilateral mixed markets. In K. Ohnishi, *Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 3* (pp. 101-126). Sharjah, UAE: Bentham Science Publishers.